Actively Dynamic Networks

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joint work with:

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Dynamic Networks

General Description:

- Networks whose structure may change
- Usually represented by a graph
- Edges and/or nodes come and go

Dynamics and Control:

- Active or Passive
- Centralized or Distributed

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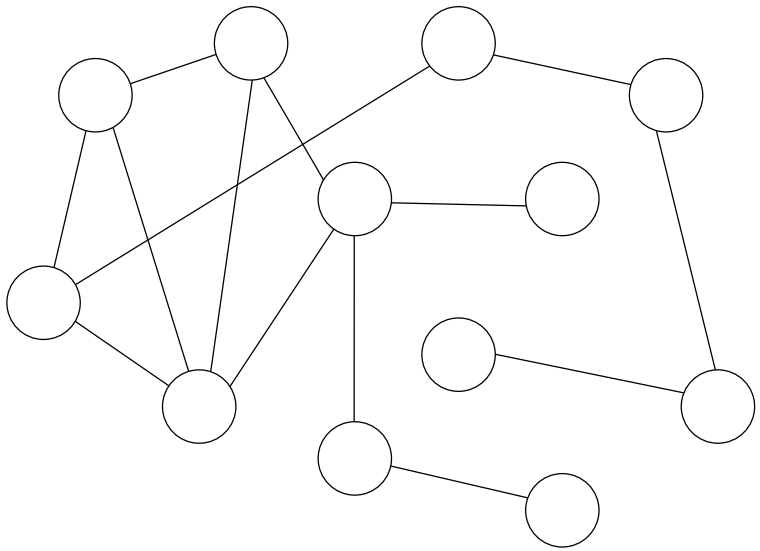

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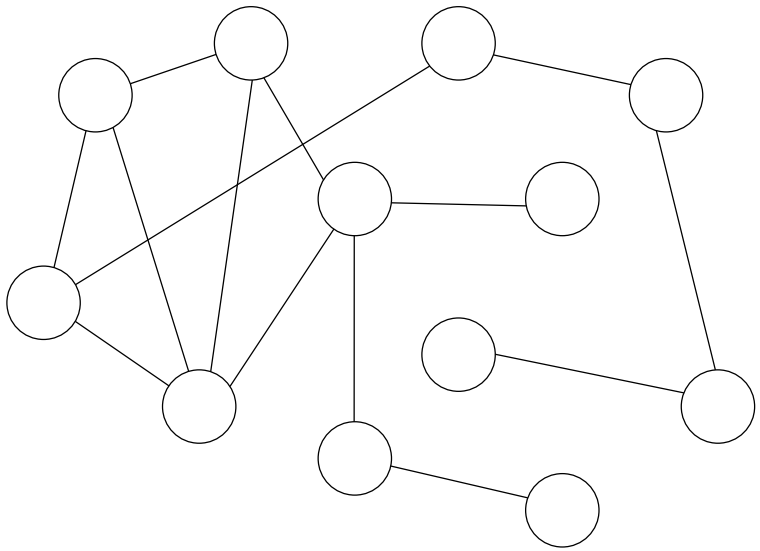

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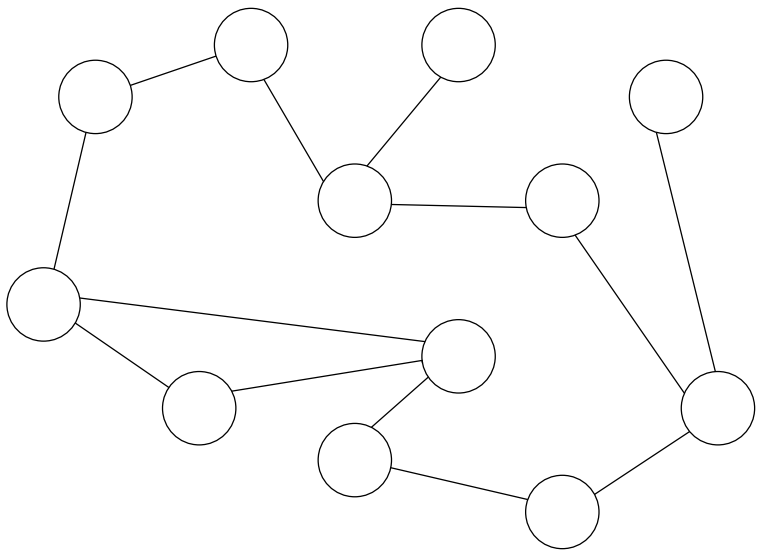

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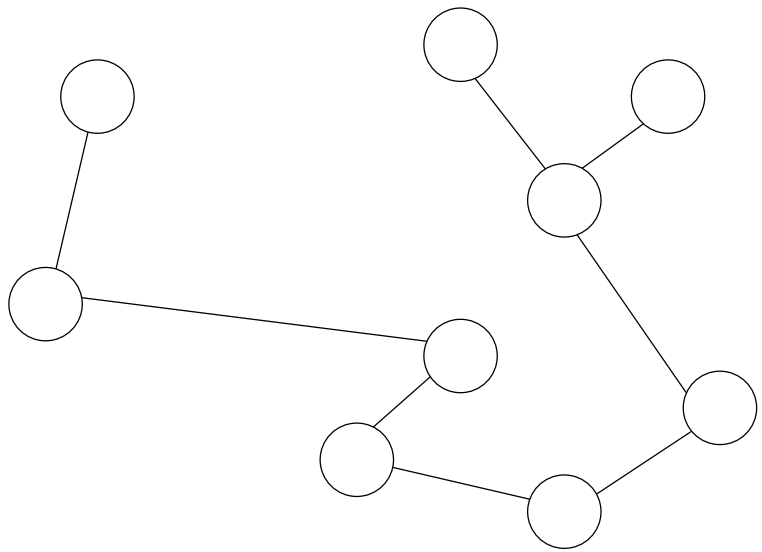

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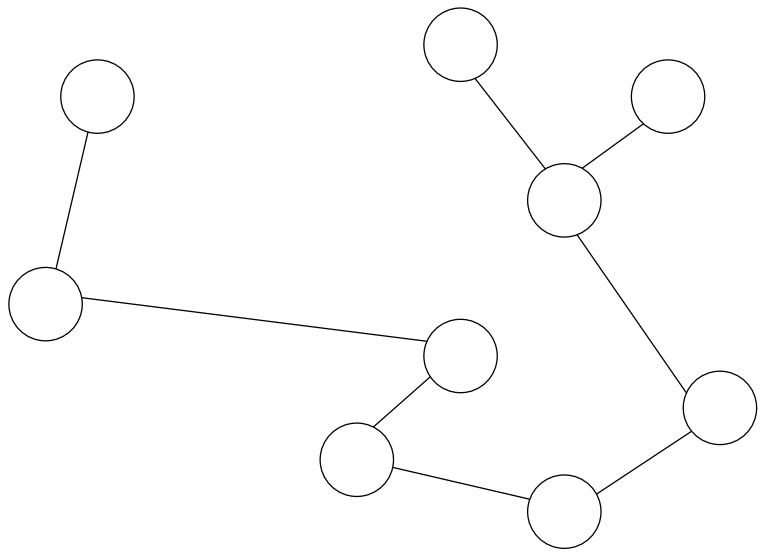

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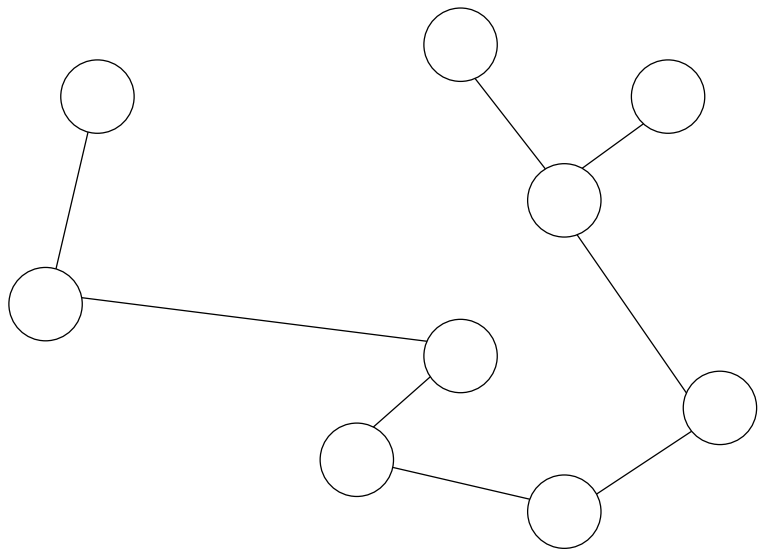

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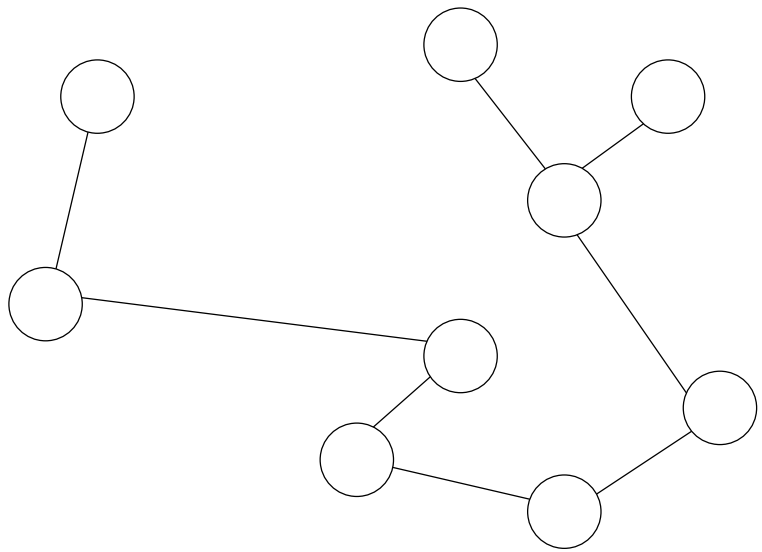

Dynamics and Control:

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- Temporal Graphs
  - Initiated by Berman [Be96] and Kempe, Kleinberg, and Kumar [JCSS00]
  - Kostakos [Ko09]
  - Multi-labeled, Mertzios, Michail, Chatzigiannakis and Spirakis [ICALP13]
  - Recent work: Molter, Renken, and Zschoche [MFCS21], Enright, Meeks, Mertzios and Zamaraev [JCSS21], Deligkas and Potapov [AAAI20], Akrida, Mertzios, Spirakis and Raptopoulos [JCSS21], Mertzios, Molter, Renken, Spirakis and Zschoche [MFCS21].

- Distributed Computation in Passively Dynamic Networks
  - Population Protocols: Angluin, Aspnes, Diamadi, Fischer and Peralta [DC06], Michail and Spirakis [DC14], Michail, Spirakis, Theofilatos [SSS18], Doty, Eftekhari, Gasieniec, Severson, Stachowiak Uznanski [FOCS21]
  - More powerful devices: Kuhn, Lynch, Oshman [STOC10], Michail, Chatzigiannakis and Spirakis [TCS11]

Related Work

- Construction of Overlay Networks
  - Stoica, Morris, Karger, Kaashoek and Balakrishnan [SIGCOMM01]: Chord (distributed hash table)
  - Aspnes and Shah [TALG07]: Skip Graphs (distributed data structure, balanced tree)
  - Construction: Angluin, Aspnes, Chen, Wu and Yi [SPAA05], Aspnes and Wu [OPODIS07], Gmyr, Hinnenthal Scheideler and Sohler, [ICALP17], Gotte, Hinnenthal and Scheideler, [SIROCCO19] Gilbert, Pandurangan, Robinson, and Trehan [PODC20]

- Programmable Matter
  - Geometry plays a big role.
  - Derakhshandeh, Dolev, Gmyr, Richa, Scheideler and Strothmann [SPAA14], Michail, Skretas and Spirakis [JCSS19], Akitaya *et al.* [ESA19]

Two Models in this Talk

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Distributed Computation and Reconfiguration

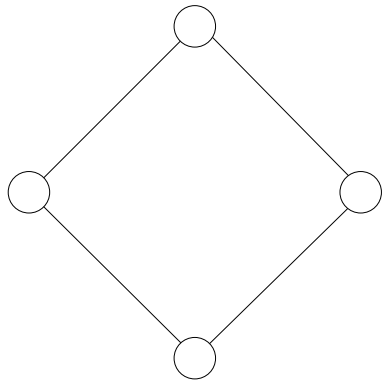
- Static set of nodes, dynamic set of edges
- Edge activations/deactivations
- Transform the initial network G_s into a target network G_f from a family of target networks
- Exploit some good properties of G_f in order to more efficiently solve a distributed task

The Complexity of Growing a Graph

- Dynamic set of nodes and edges
- Node generations and edge activations/deactivations
- Construct an input graph G starting from graph G_0 (graph with a single node) via node generations and edge activations

The First Model

- Initial connected network G_s
- Static set V of n nodes
- Dynamic set $E(i)$ of m active edges
- Standard synchronous message passing model
- Each node has a unique ID
- Nodes can only compare unique IDs
- Node u can activate an edge with node v if they have a common neighbor
- One edge between two nodes



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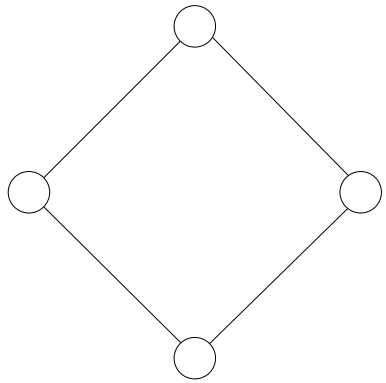

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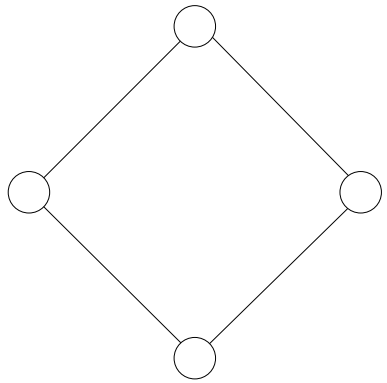

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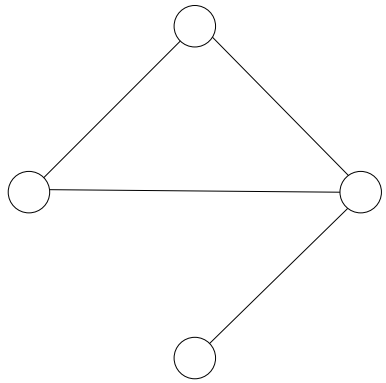

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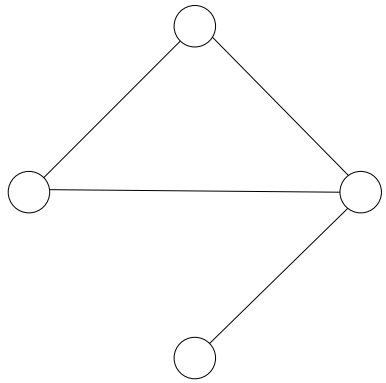

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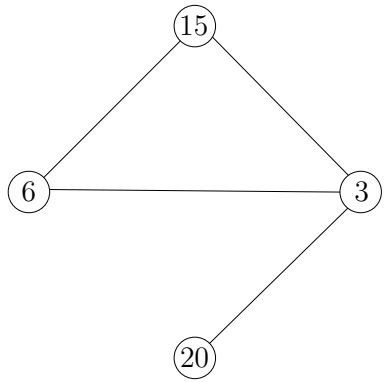

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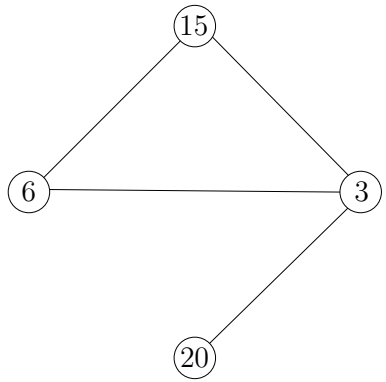

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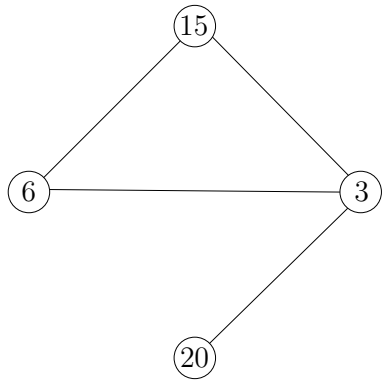

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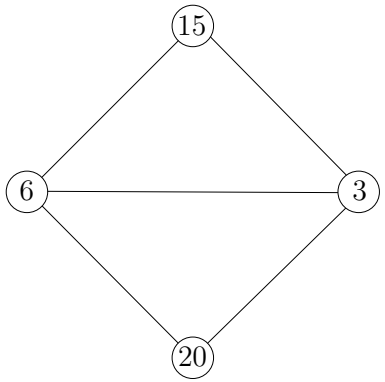

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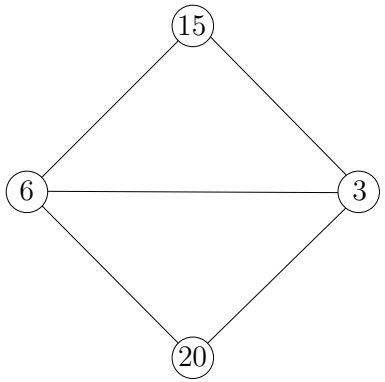

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Problems:

- Leader Election: Elect a unique leader
- Token Dissemination: Each node has a unique piece of information. Every piece of information has to be disseminated to every node
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Very simple strategy:

- Initial Cycle network, target Star network
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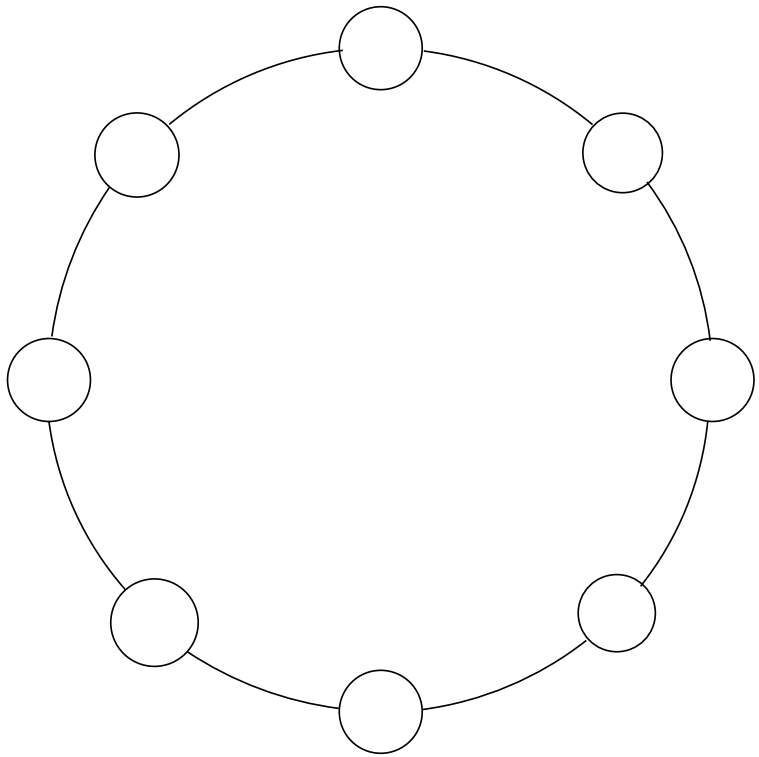

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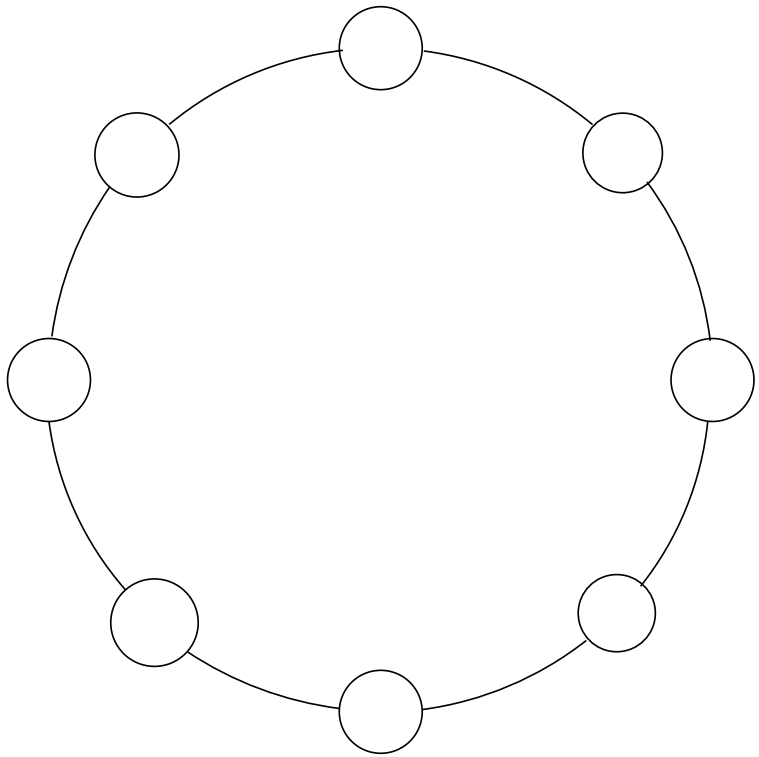

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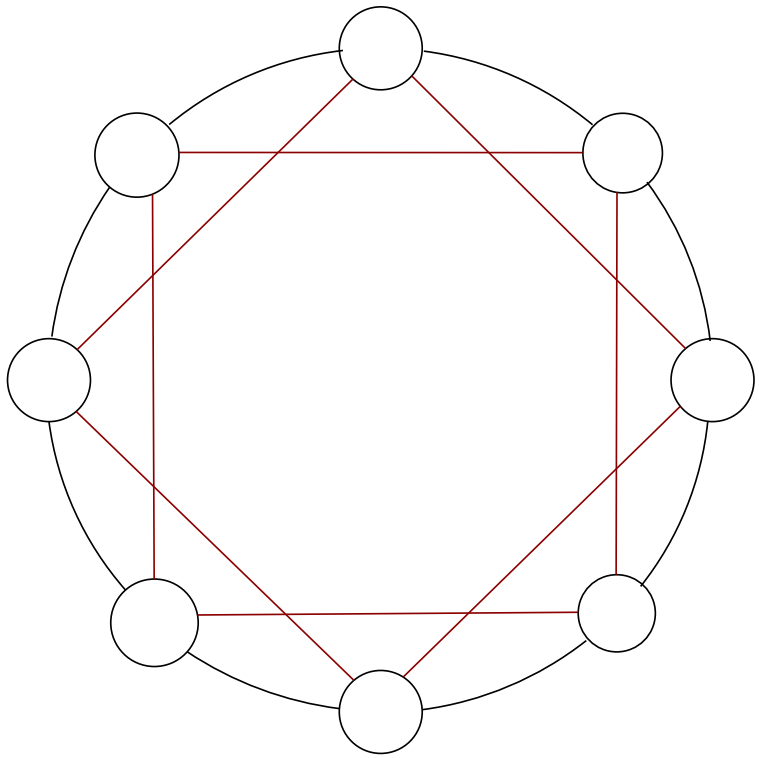

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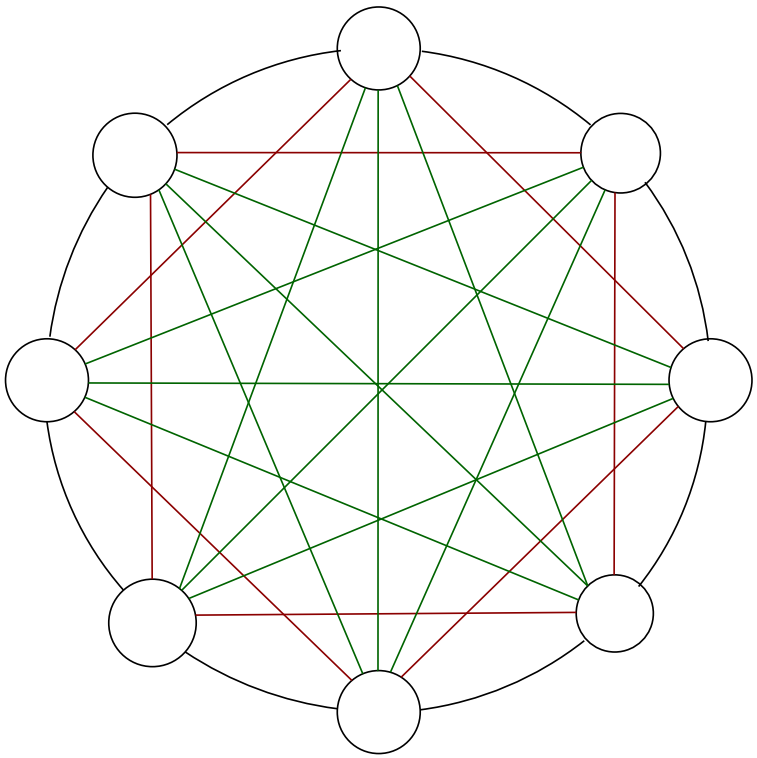

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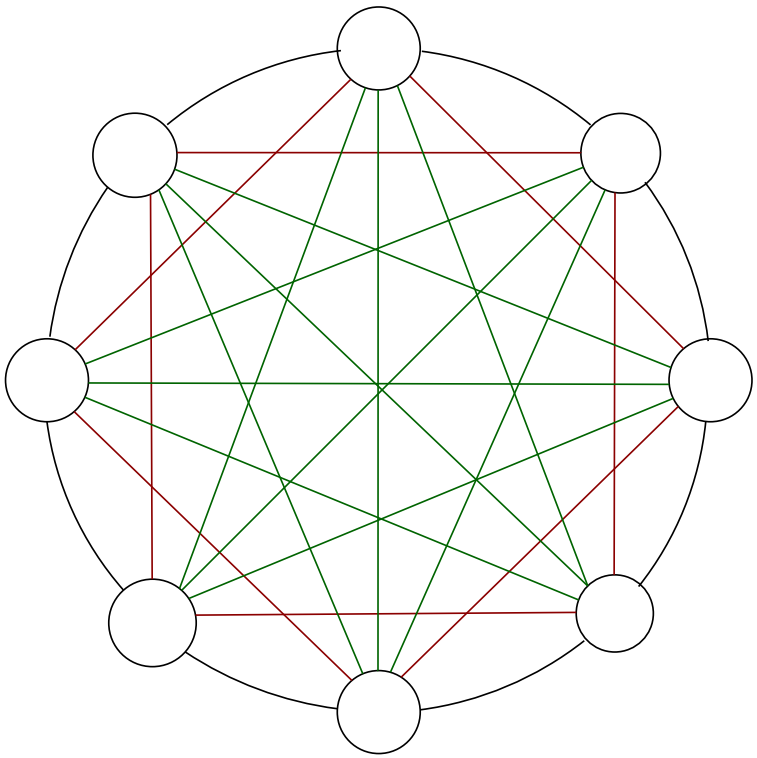

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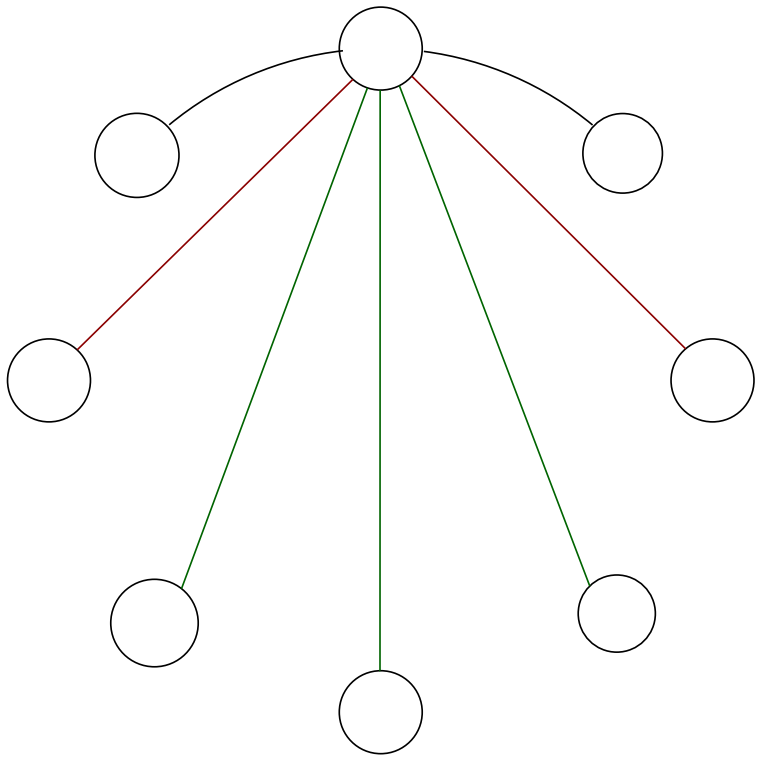

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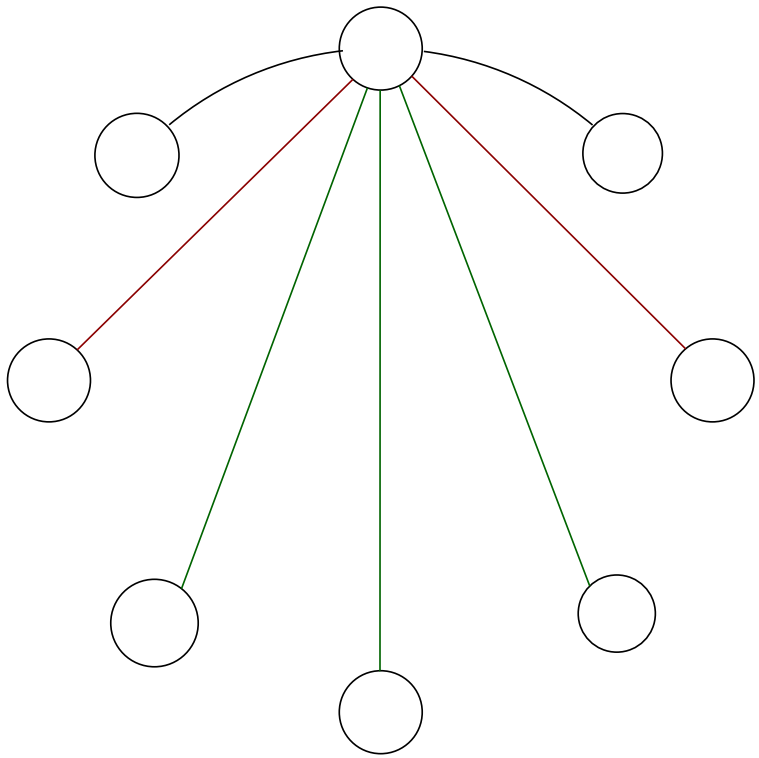

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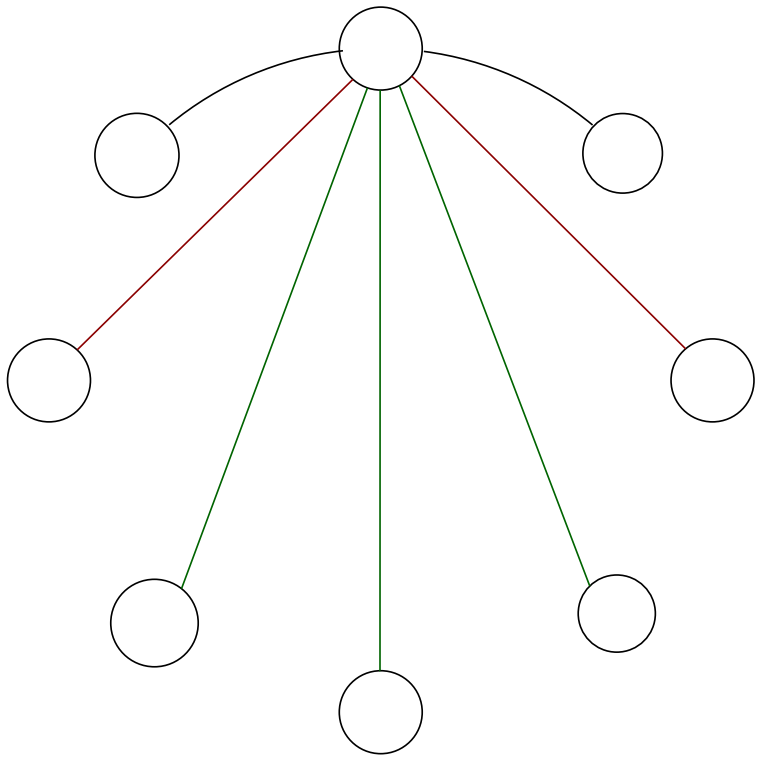
- Activating and maintaining a connection does not come for free



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- Total Edge Activations: The number of edges activated by the algorithm
- Maximum Activated Edges: The maximum number of activated edges by the algorithm per round
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Objective

Give (poly)log(n) time algorithms for the general task of transforming any G_s into a G_f of diameter (poly)log(n), while minimizing the edge-complexity, and solve leader election, depth-(poly)log(n) tree, and token dissemination at the same time.

Our Results

Algorithm	Time	Total Edge Activations	Maximum Activated Edges	Maximum Activated Degree
GraphToStar	$O(\log n)$	$O(n \log n)$	$O(n)$	$O(n)$
GraphToWreath	$O(\log^2 n)$	$O(n \log^2 n)$	$O(n)$	$O(1)$
GraphToThinWreath	$O(\log^2 n / \log \log n)$	$O(n \log^2 n)$	$O(n)$	$O(\log^2 n)$

Table: Algorithms

Bounds	Time	Total Edge Activations	Maximum Activated Edges	Maximum Activated Degree
Centralized Lower	$\Omega(\log n)$	$\Omega(n)$	$\Omega(n/\log n)$	
Centralized Upper	$O(\log n)$	$\Theta(n)$		
Distributed Lower	$O(\log n)$	$\Omega(n \log n)$		

Table: Bounds for Depth-d tree

Introduction on the General Strategy

- Nodes are partitioned into committees
- Committees organized into gadget networks
- Each node forms its own committee
- Committees compete and merge
- Final network has a single committee
- Polylogarithmic running time
- Time: phases*gadgetdiameter

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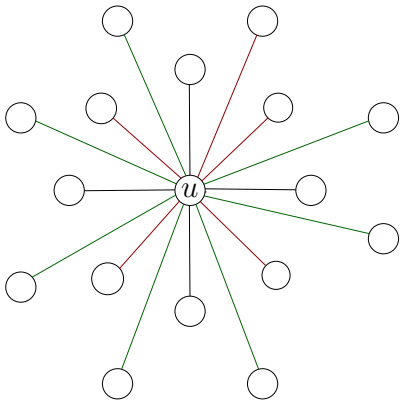

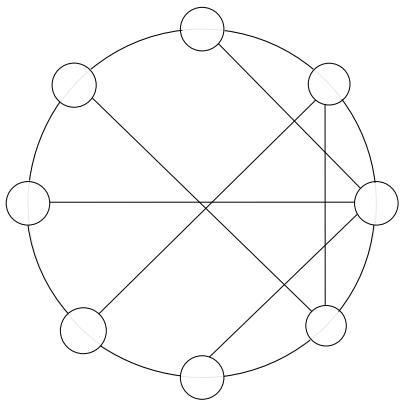

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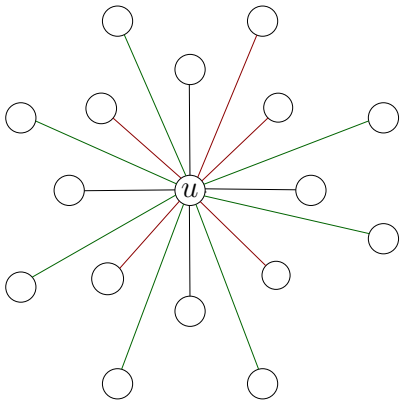

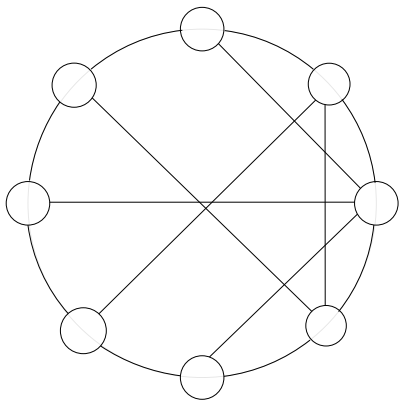

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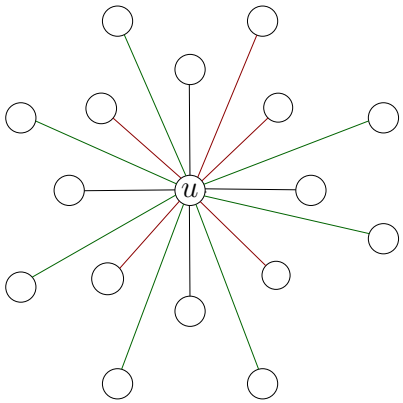

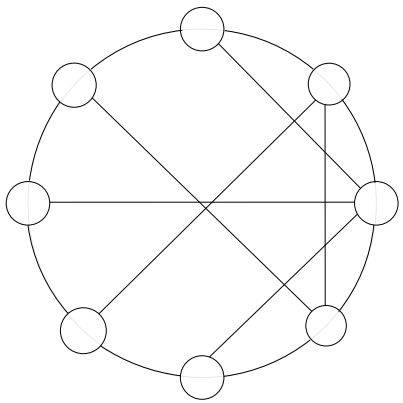

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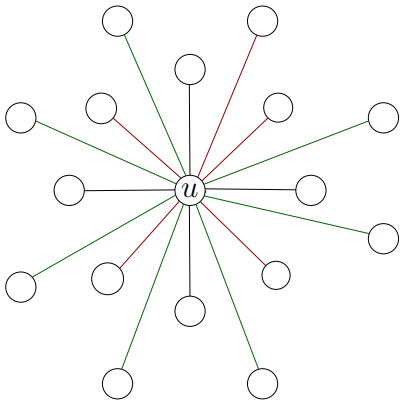

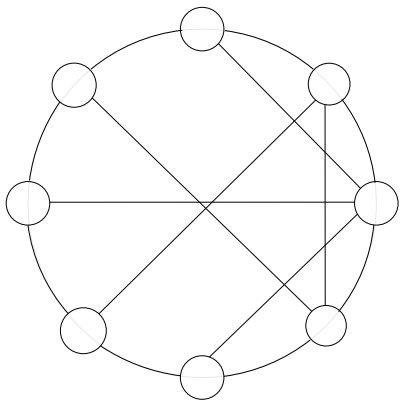

Introduction on the General Strategy

- Nodes are partitioned into committees
- Committees organized into gadget networks
- Each node forms its own committee
- Committees compete and merge
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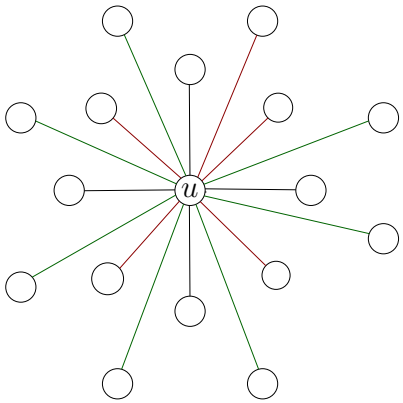

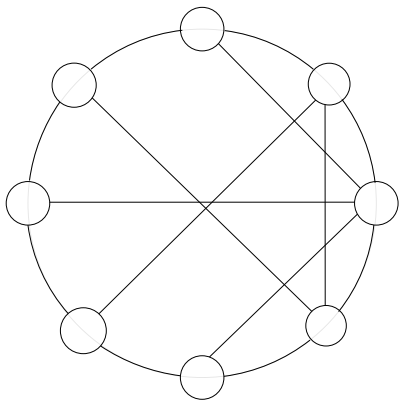

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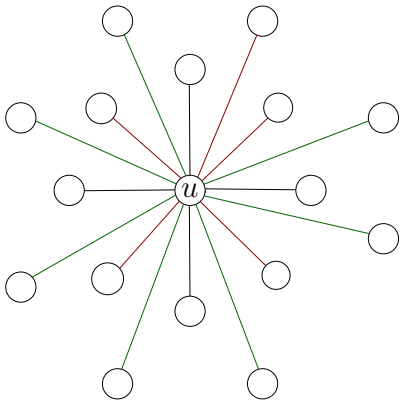

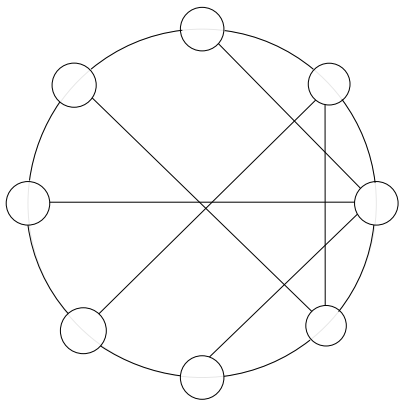

GraphToStar Algorithm

- Gadget network: Star
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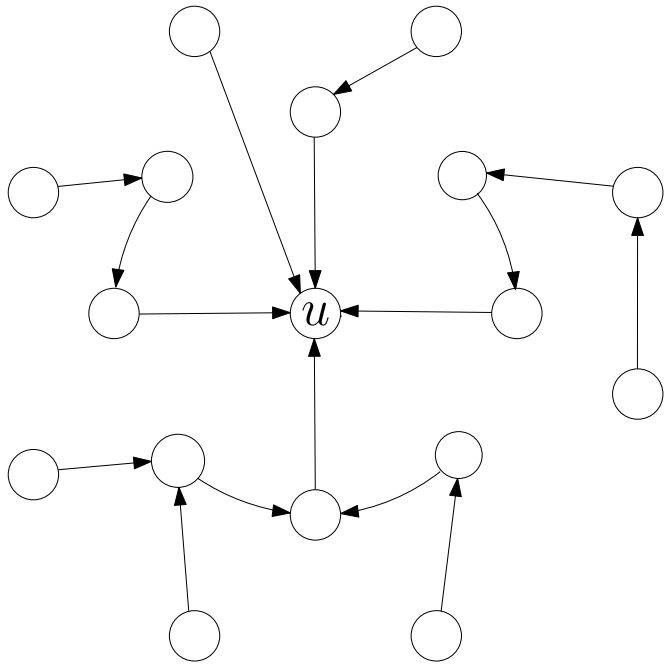

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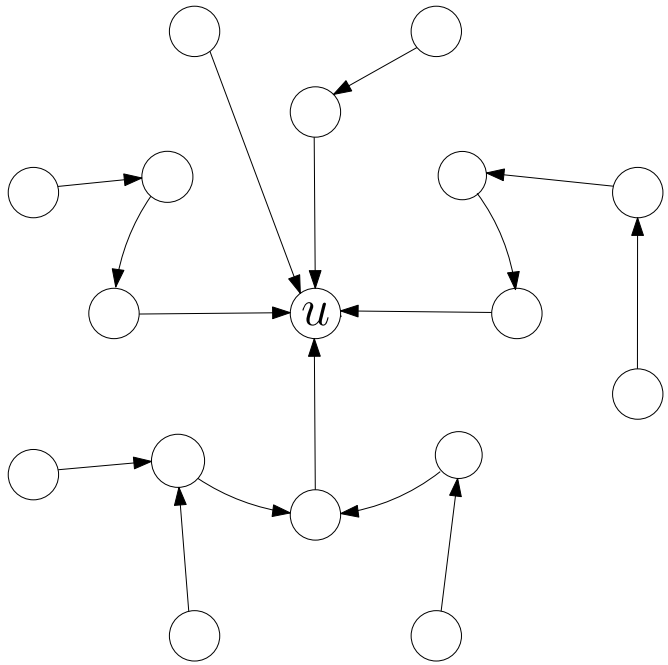

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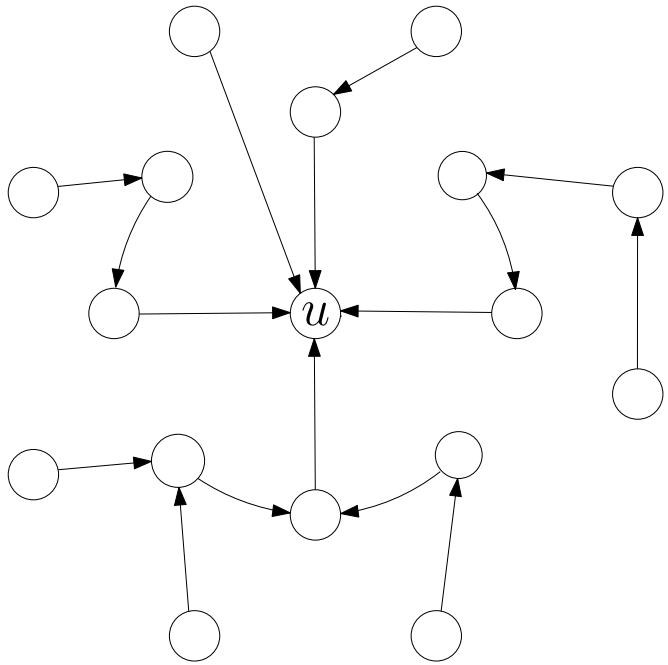

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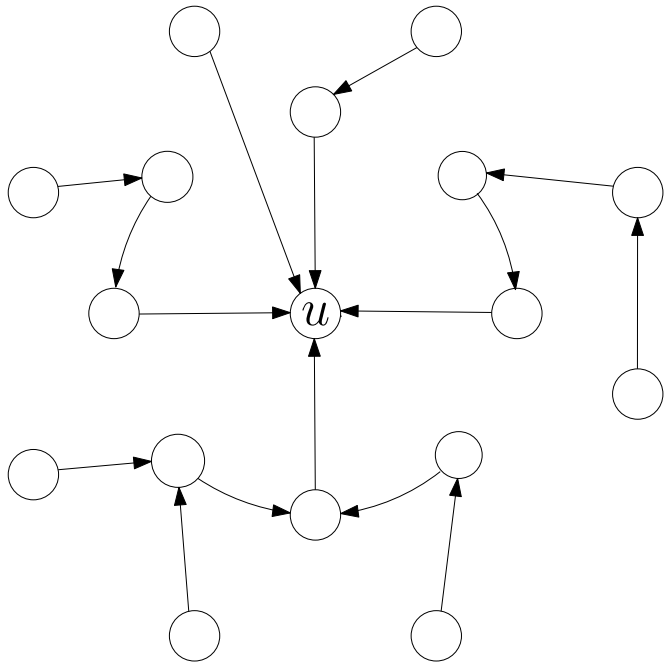

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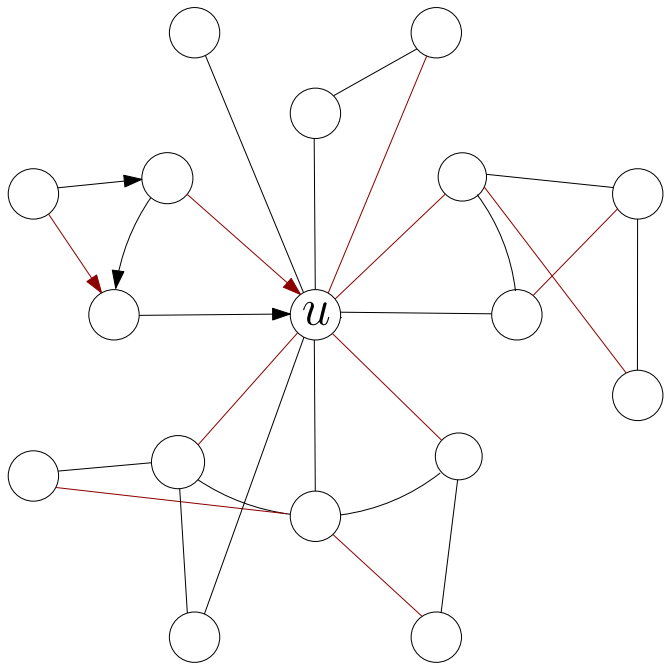

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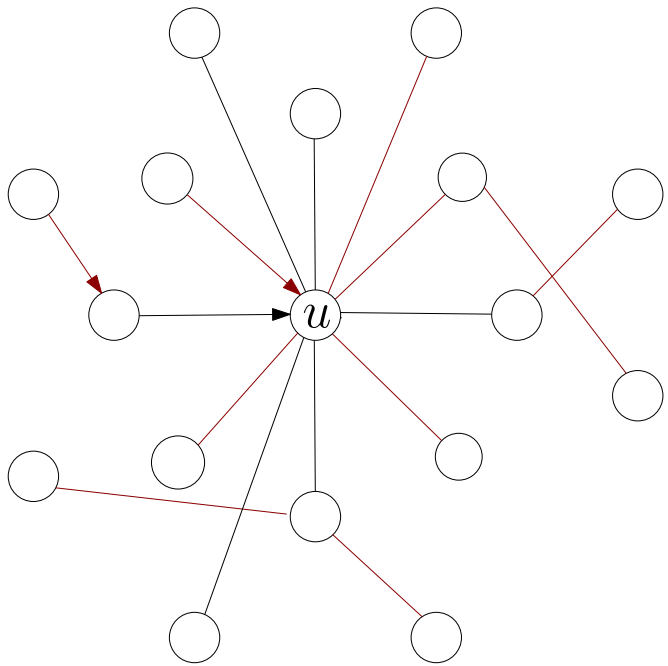

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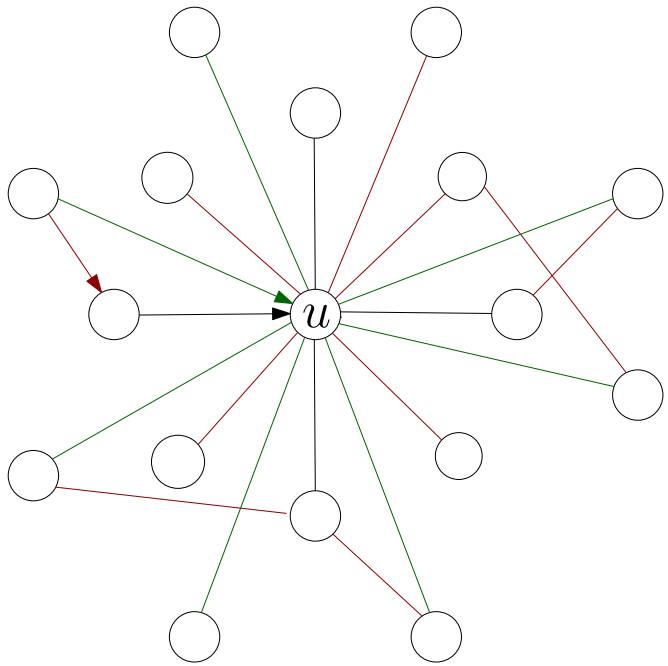

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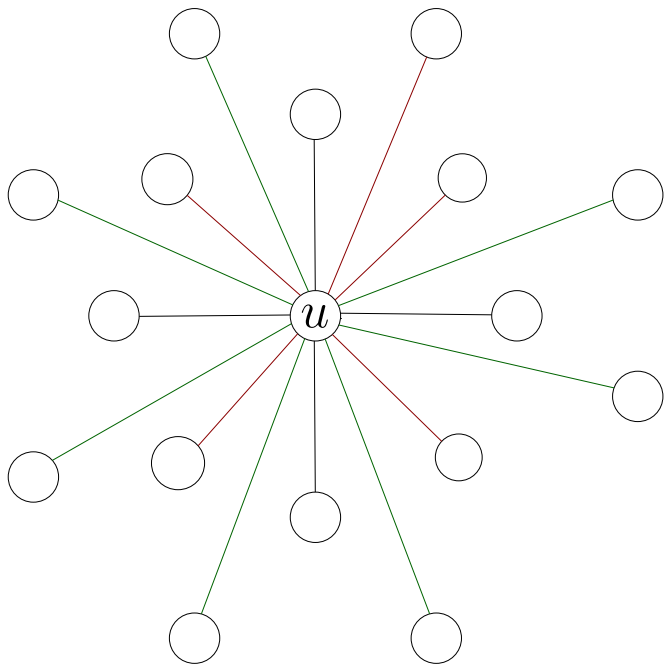

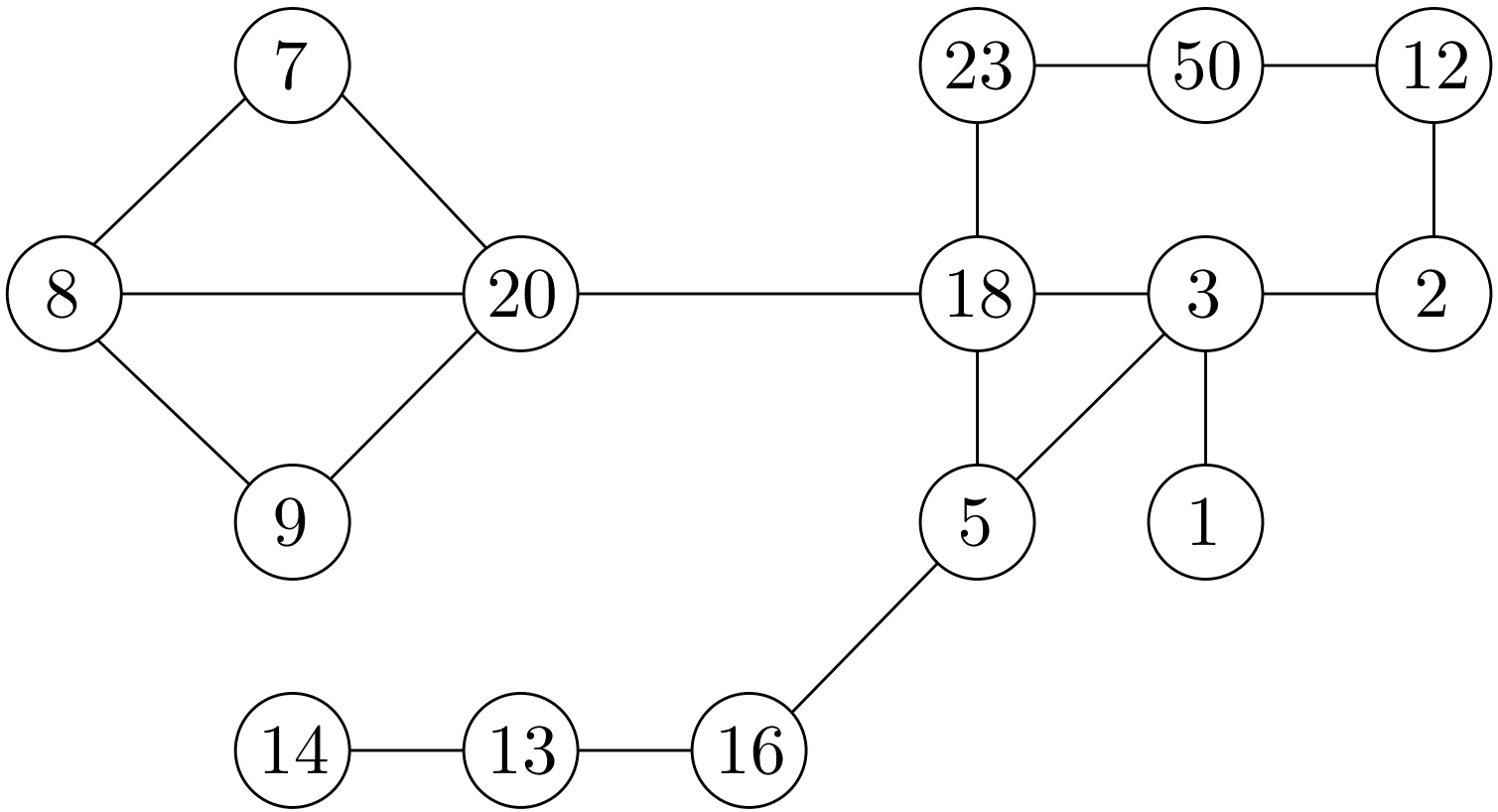
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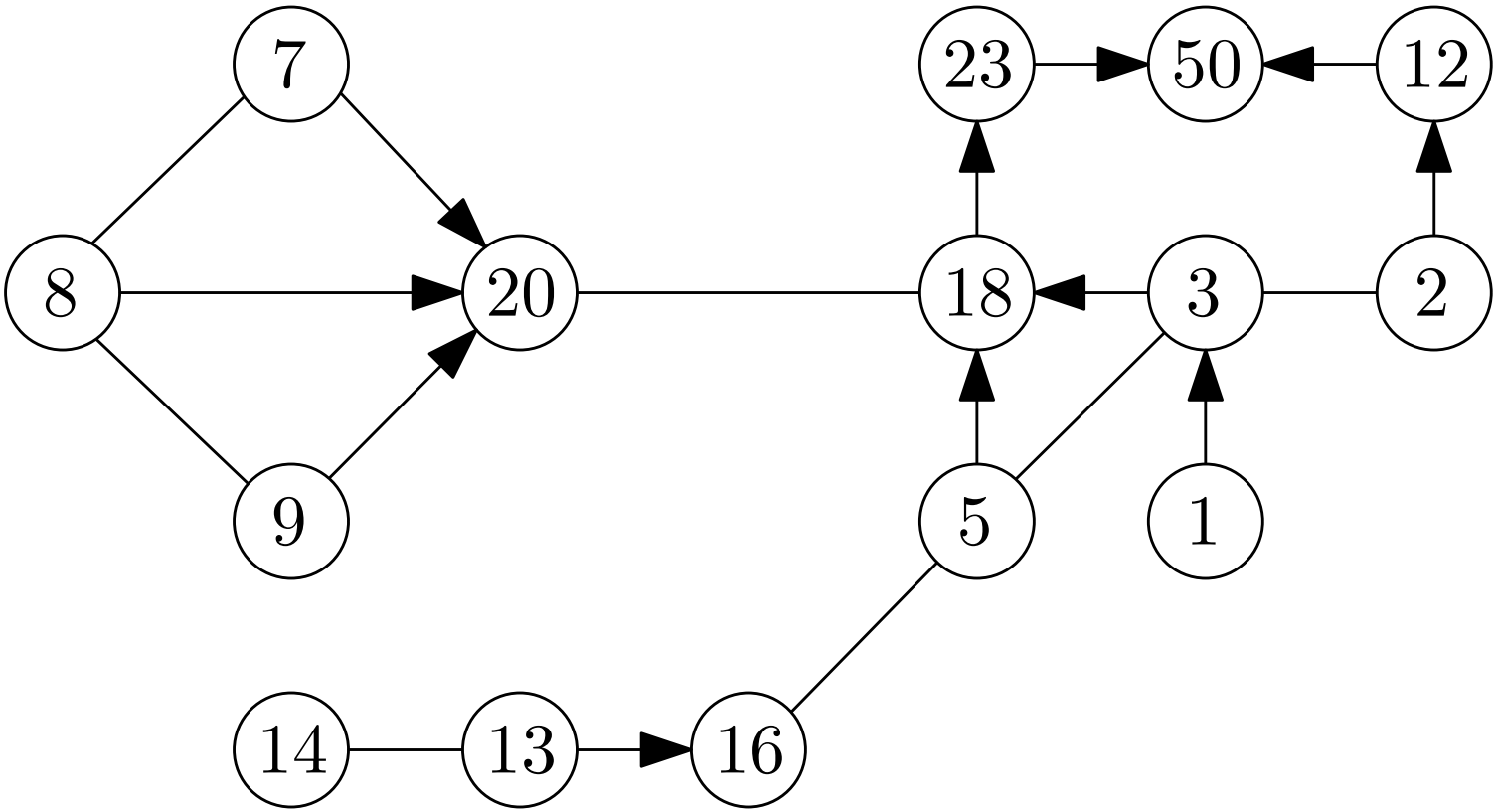
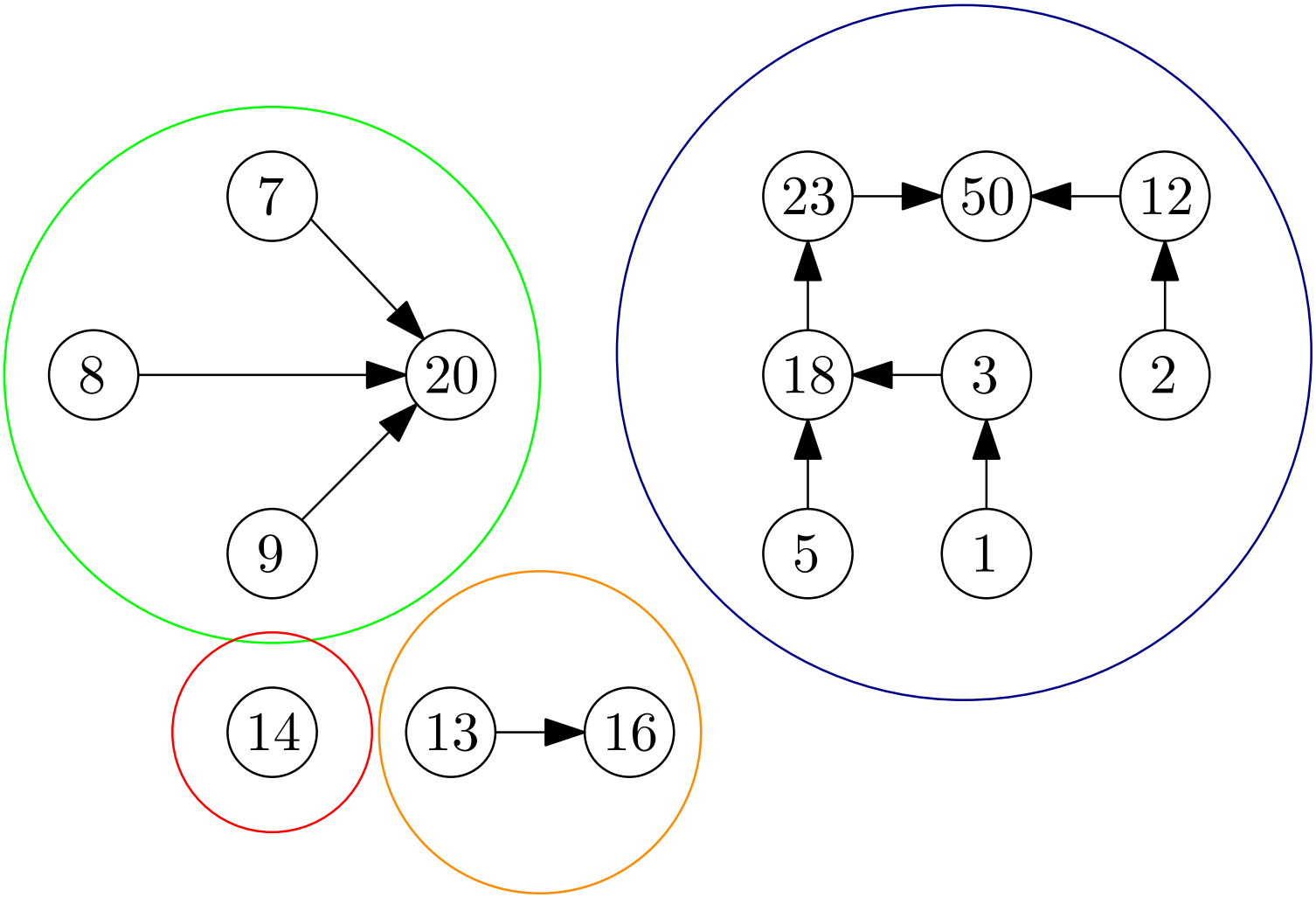
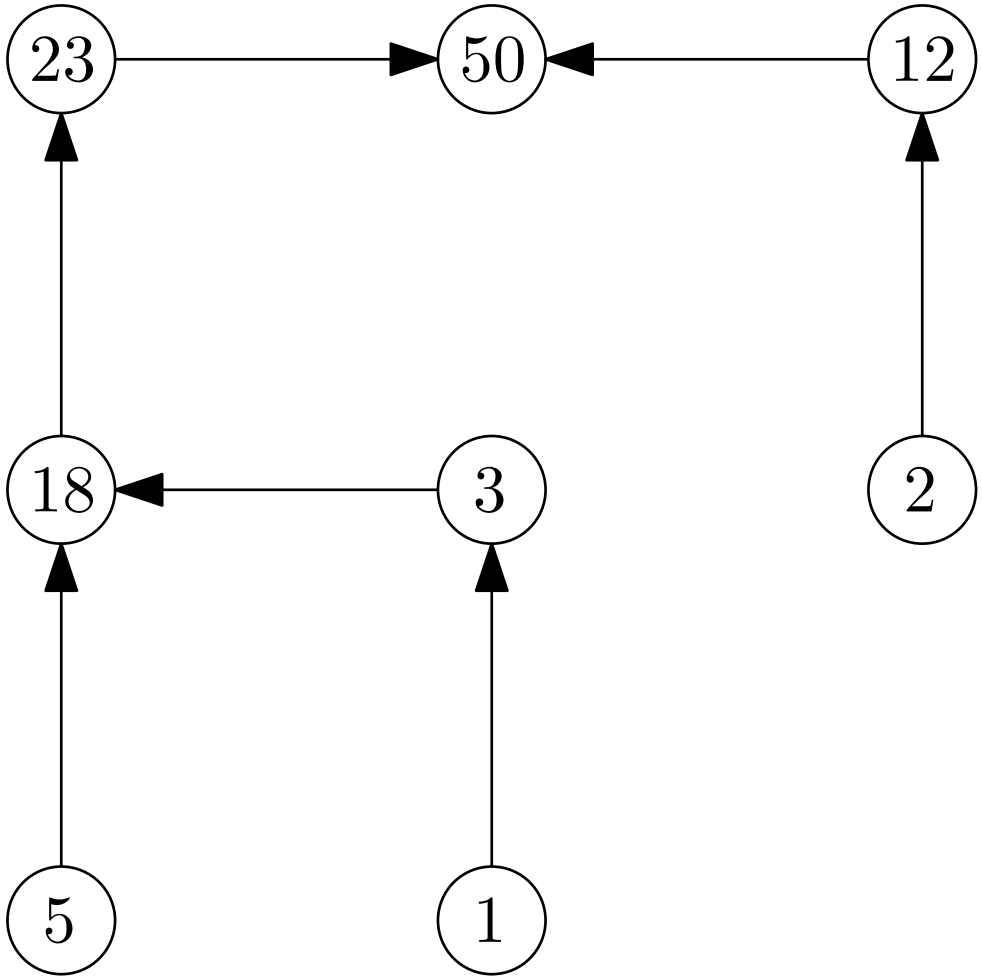


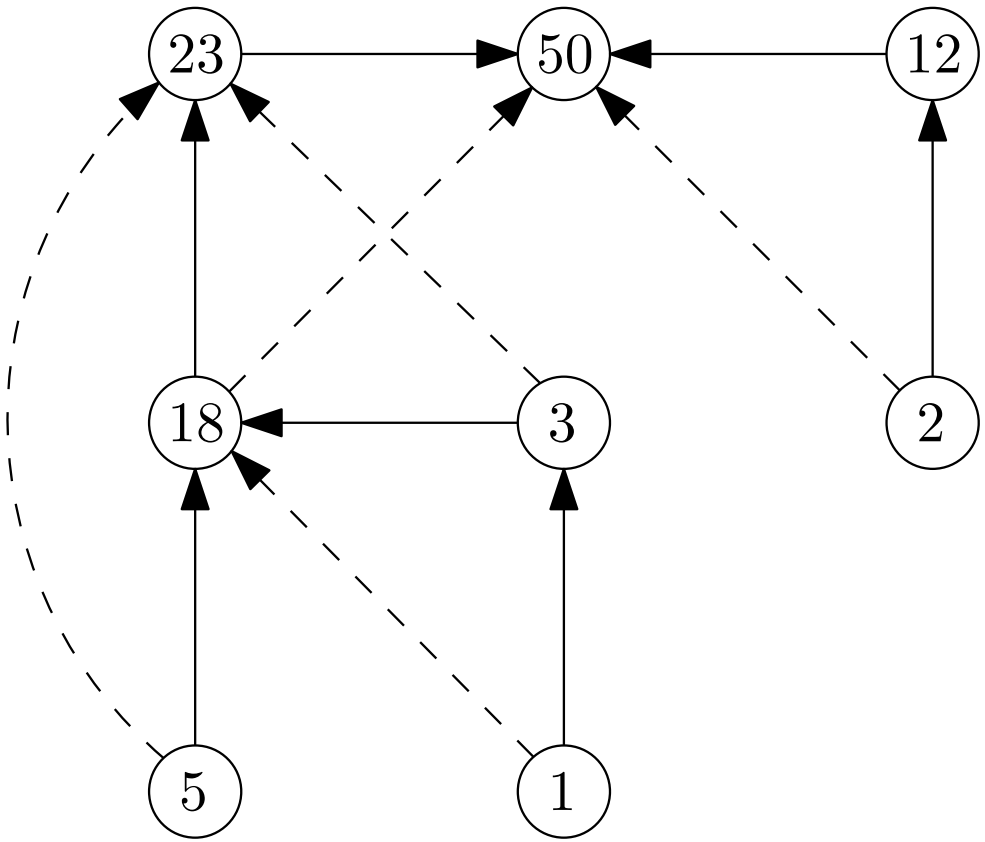
- **Selection:** Each committee leader u selects the neighboring committee leader v with the highest UID, where $UID_v > UID_u$.



- **Partitioning:** We can partition the graph into directed trees



- **Pulling:** Each committee leader u activates an edge with the next committee leader v.



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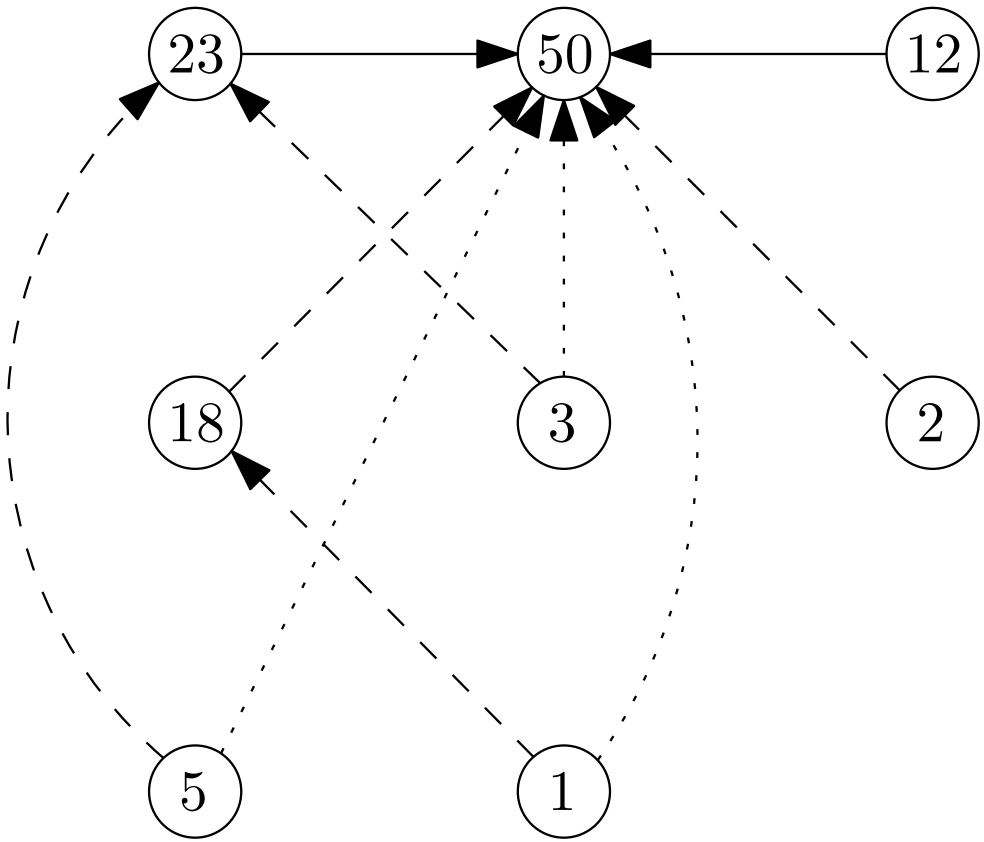

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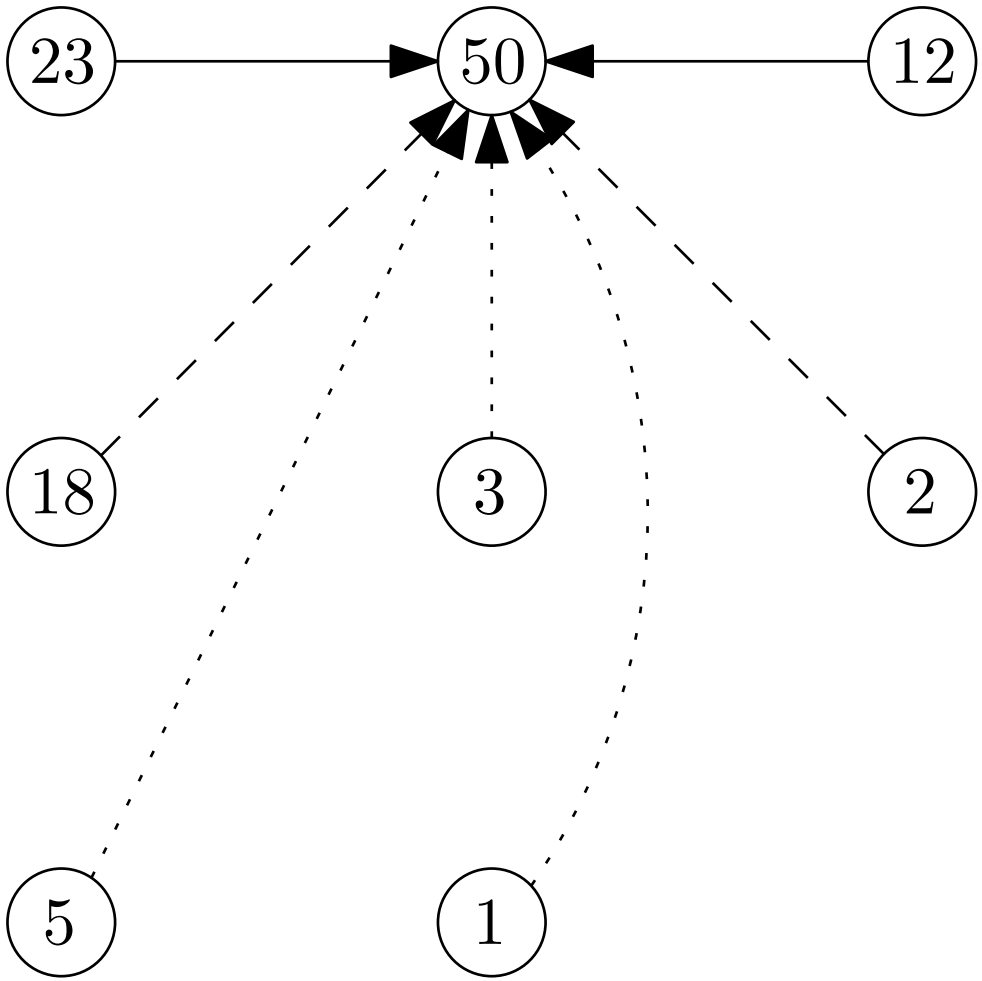

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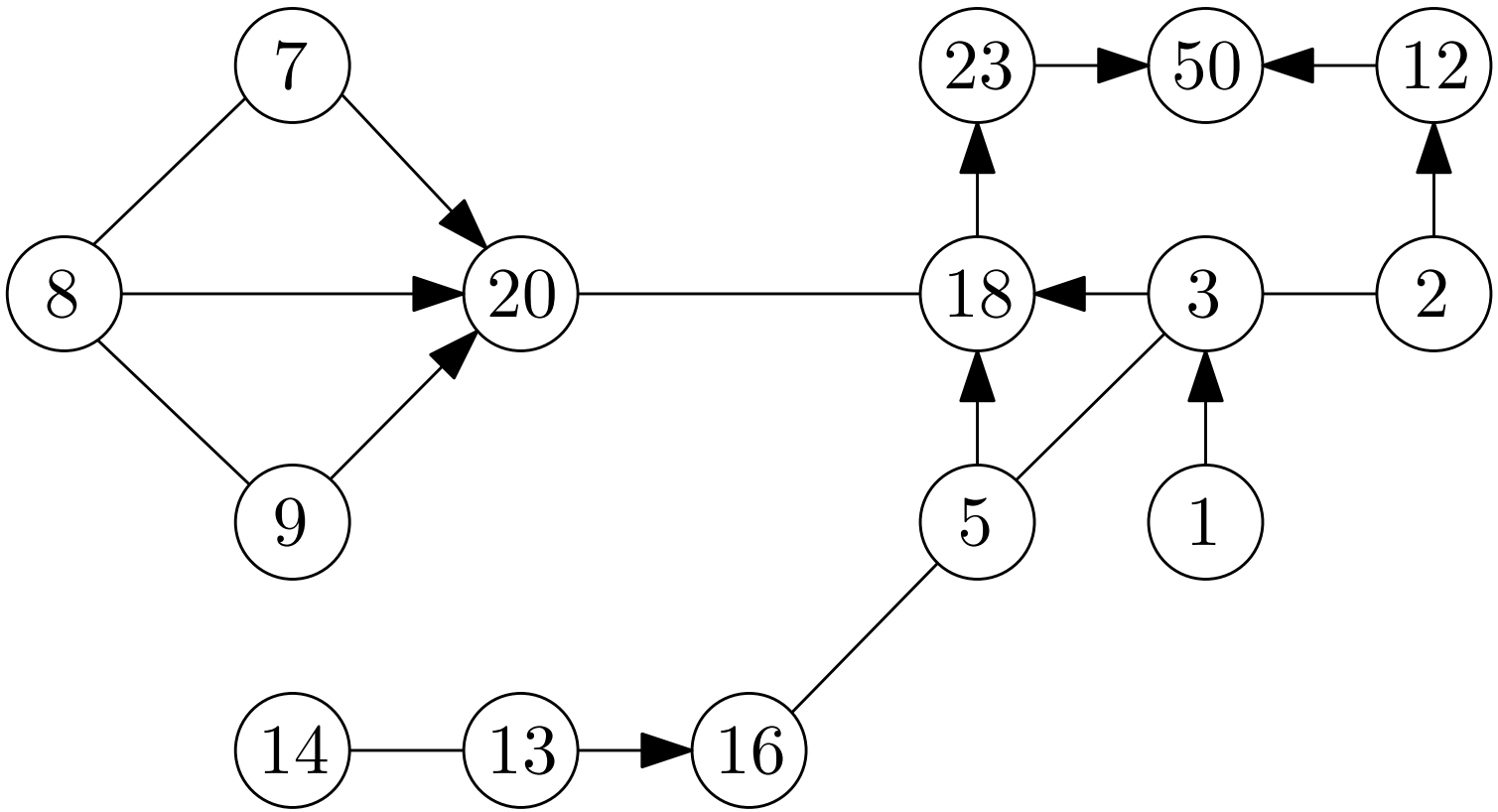
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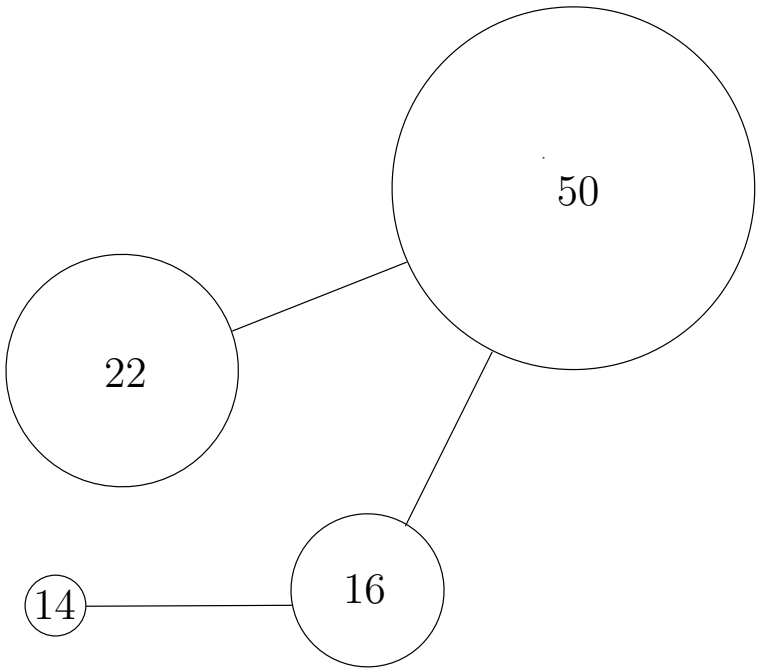
High Level



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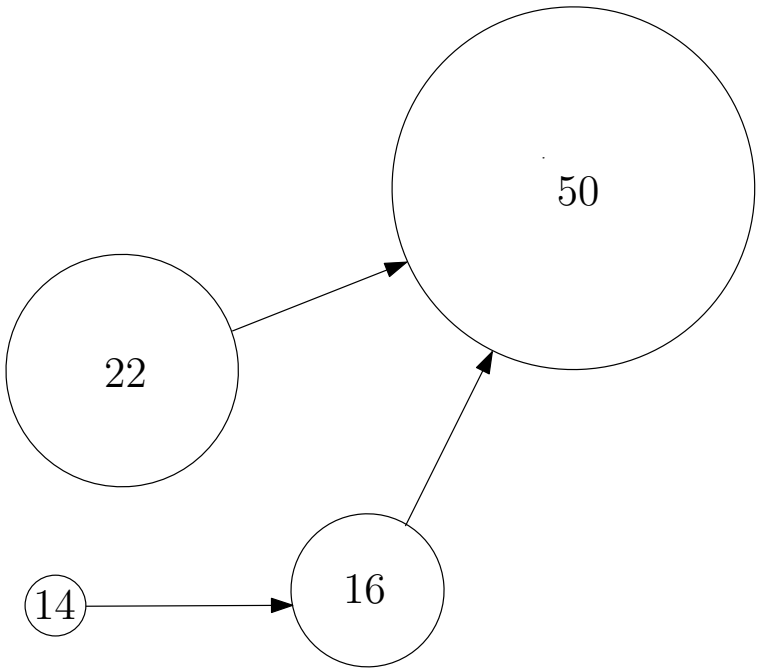

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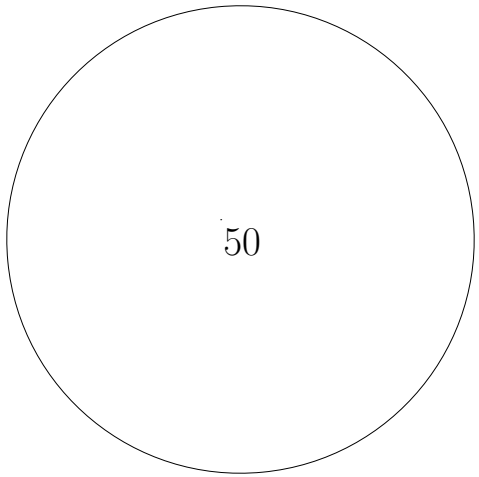

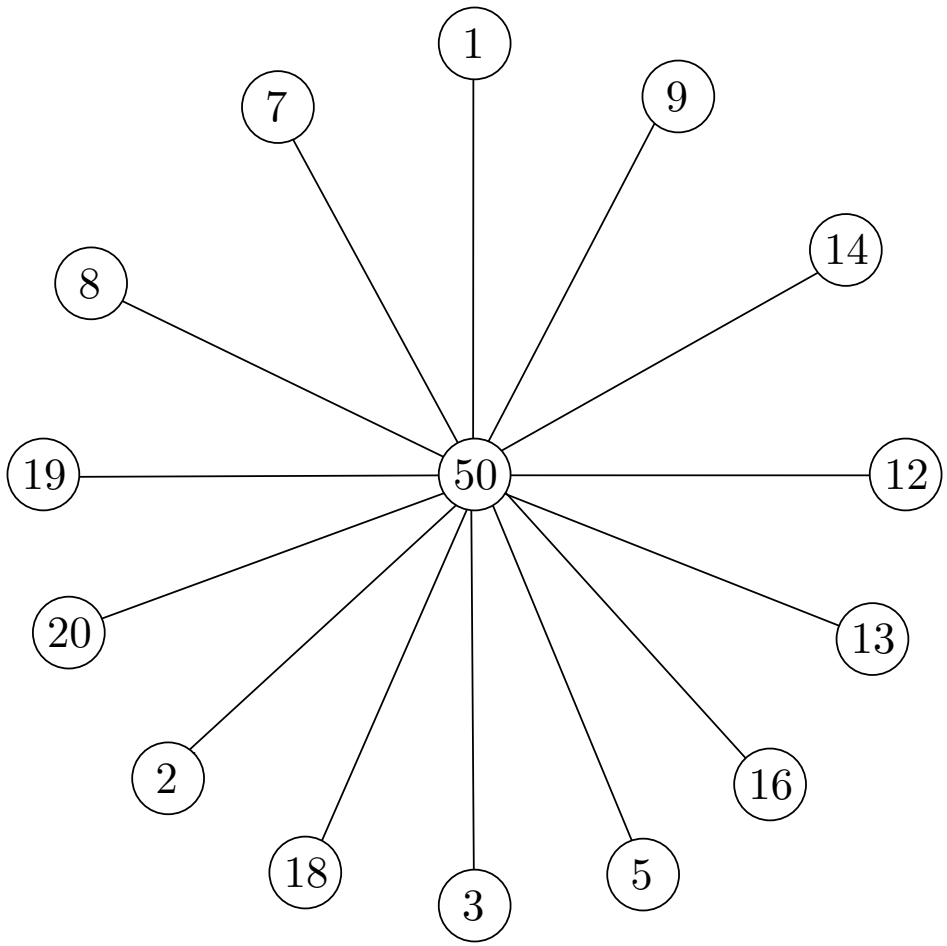
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GraphToStar Proof Sketch

Theorem: For any initial connected graph G_s, the GraphToStar algorithm solves the Depth-1 Tree problem in $O(\log n)$ time with at most $O(n \log n)$ total edge activations and $O(n)$ active edges per round

Correctness

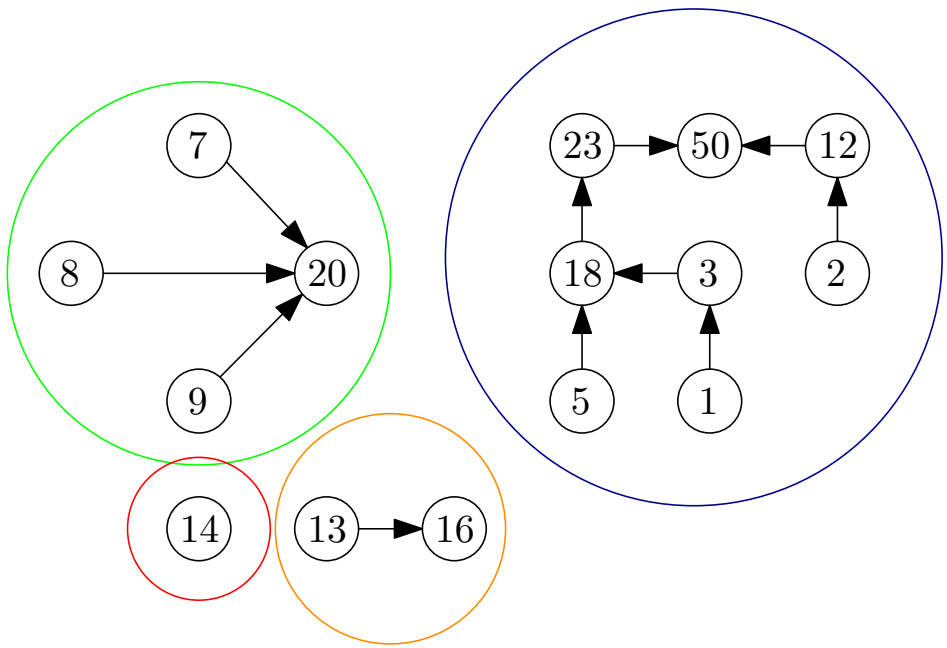
- Committees keep merging
- Can't get isolated indefinitely

Time Complexity

- Committees "double" in size
- Isolated

Edge Complexity

- Very simple



GraphToWreath Algorithm

- Gadget network: Wreath
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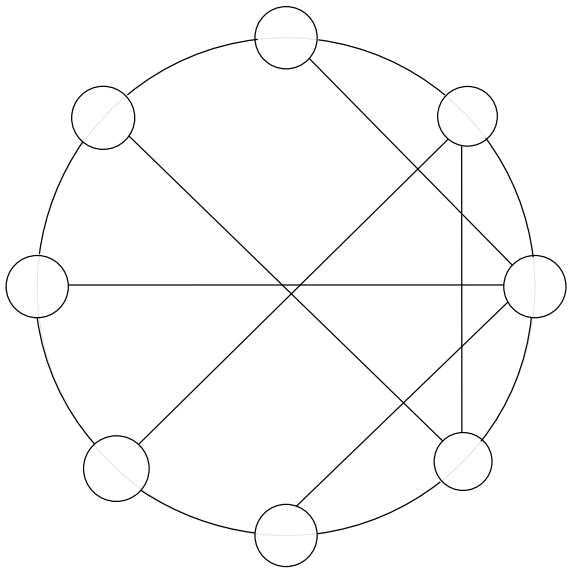

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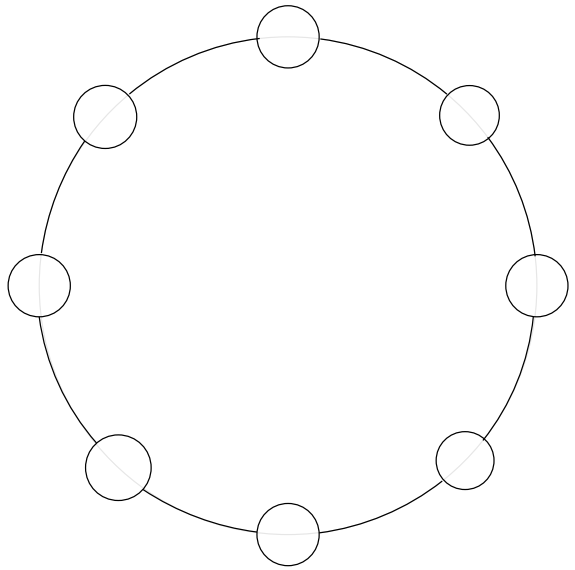

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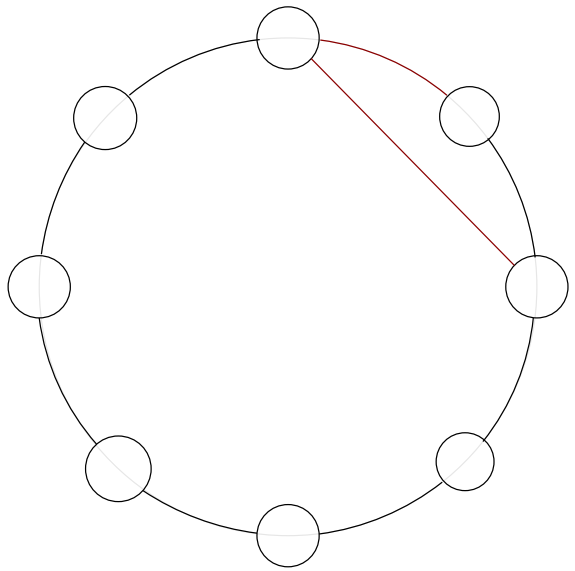

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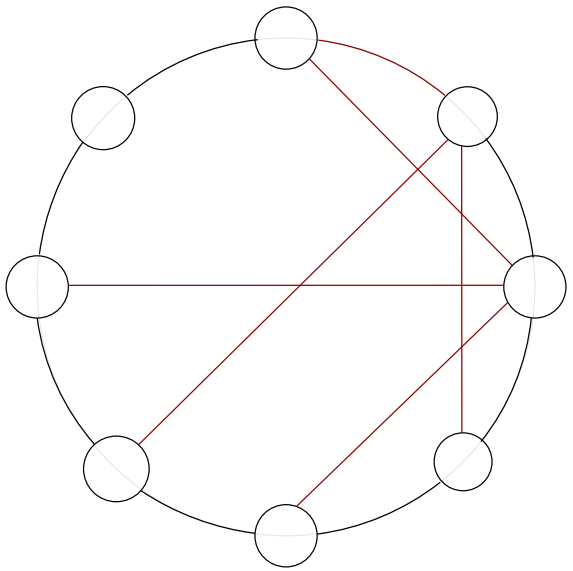

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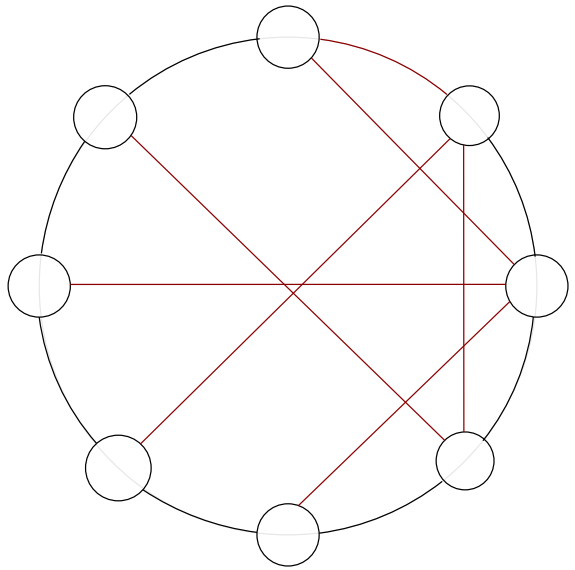

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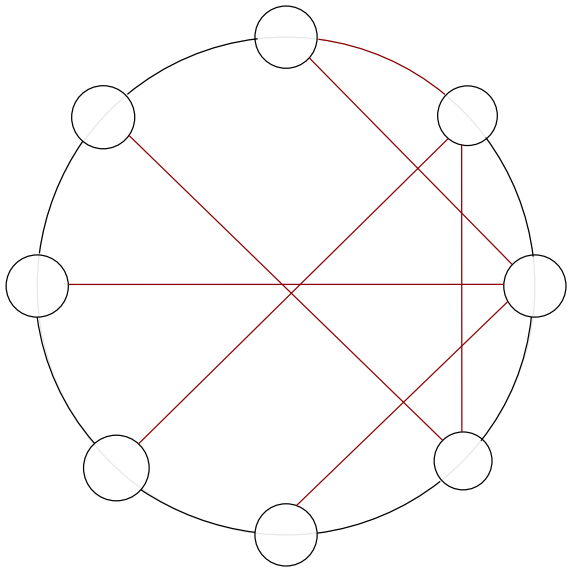

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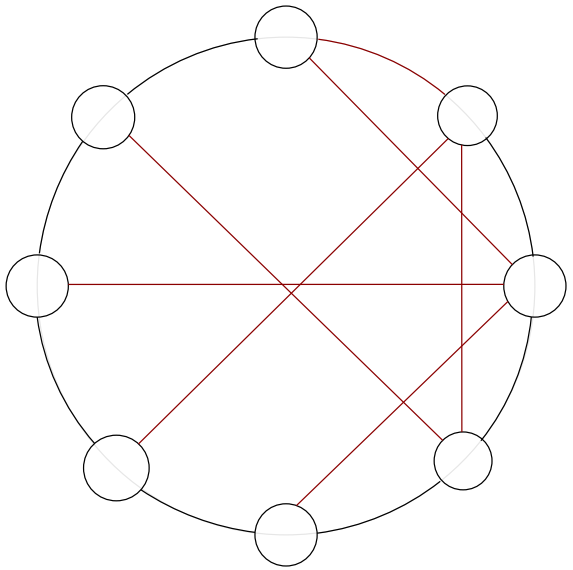

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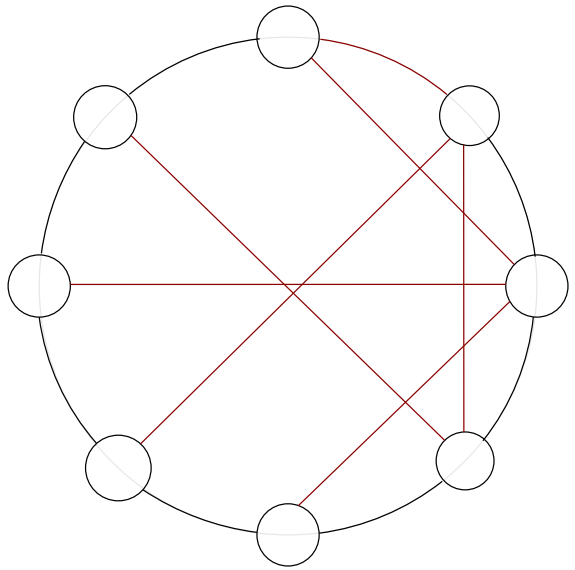

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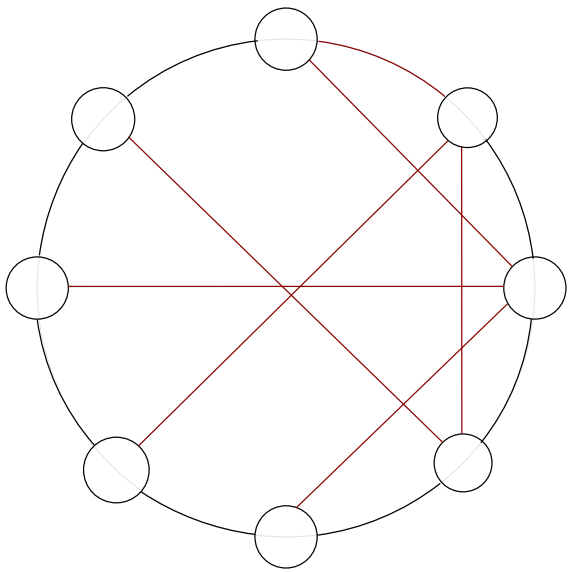

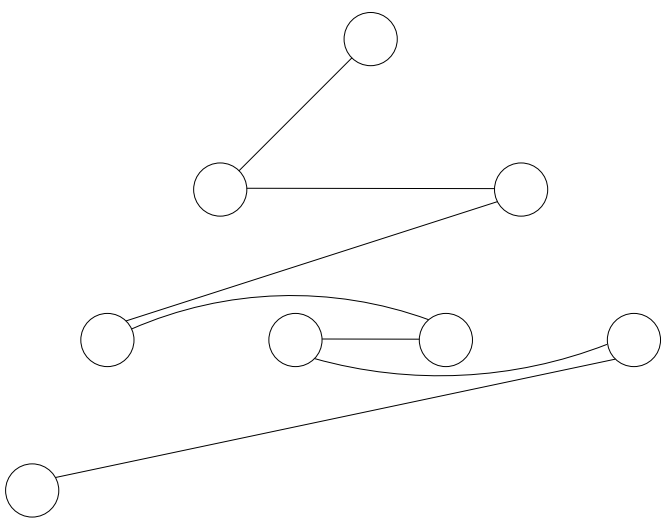

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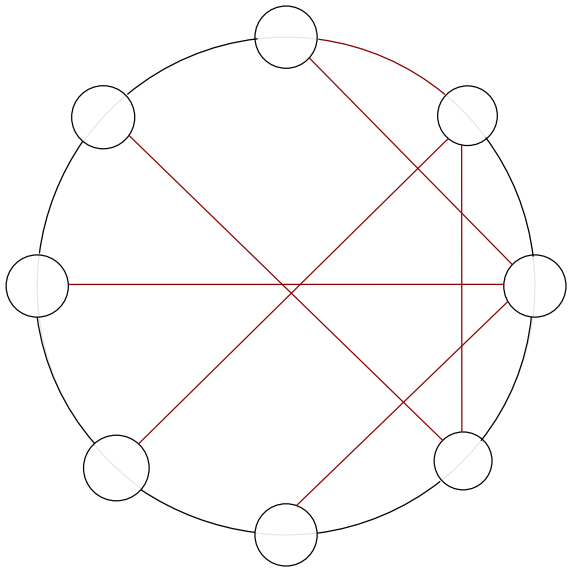

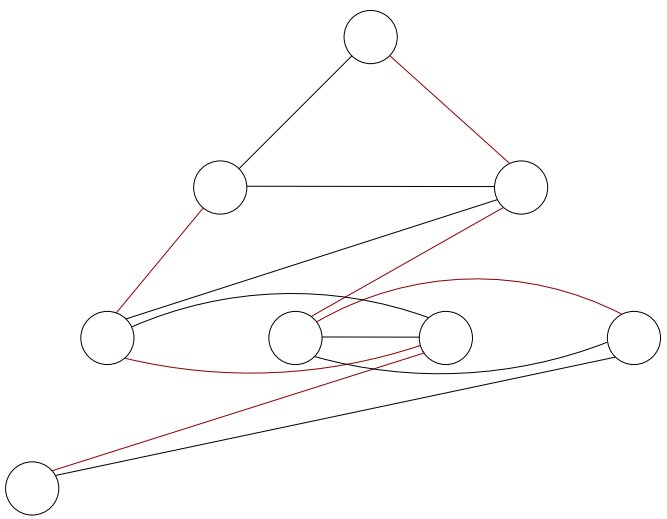

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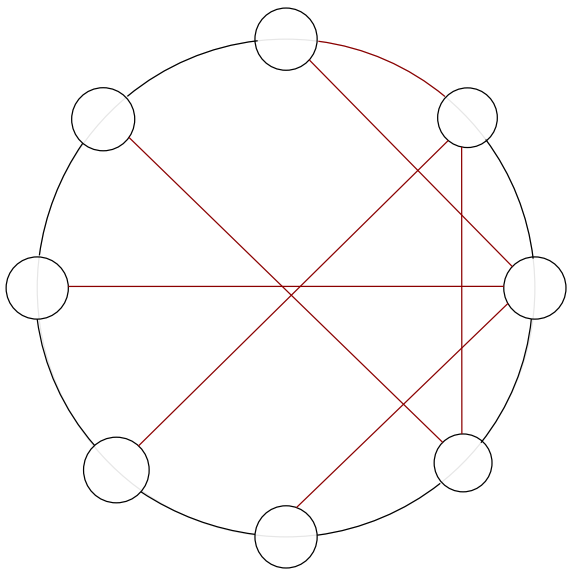

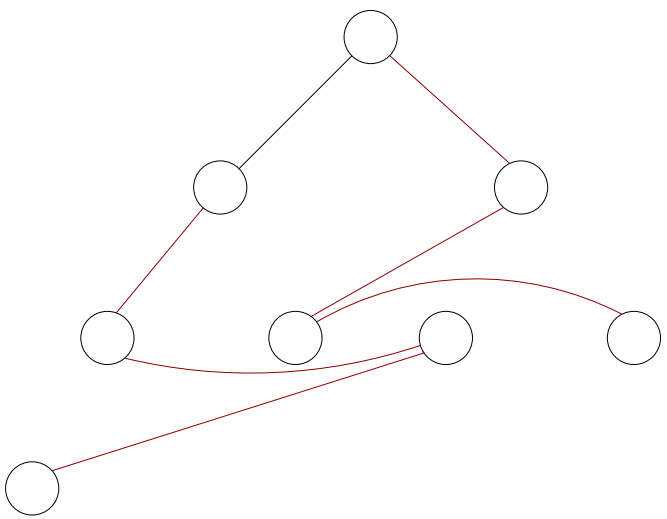

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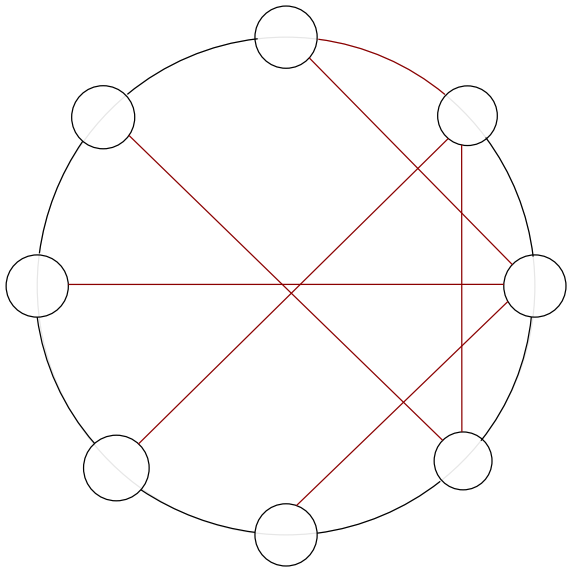

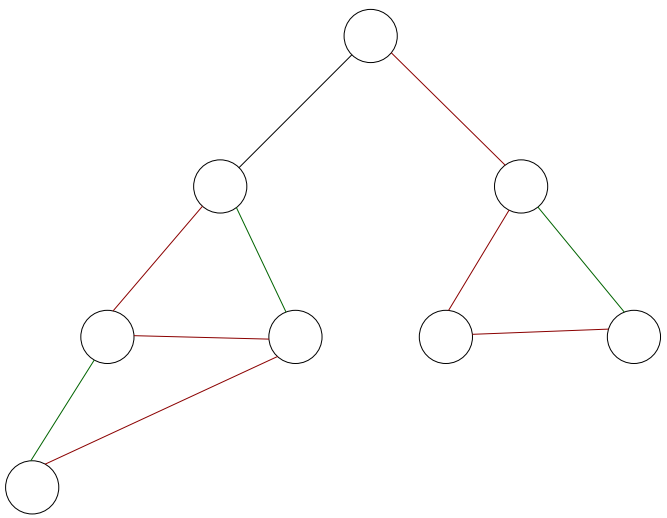

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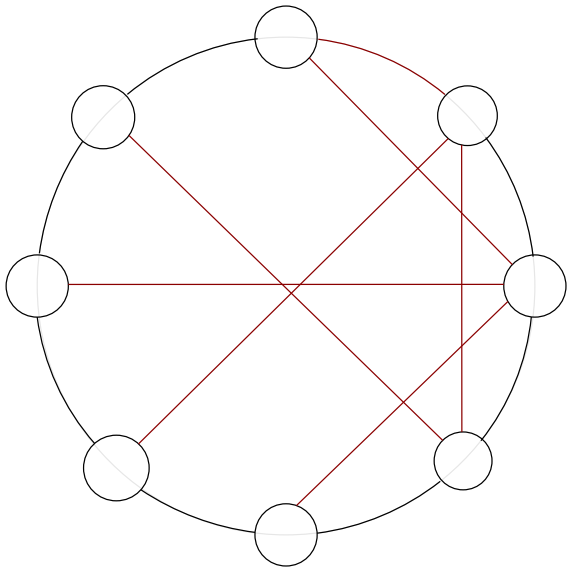
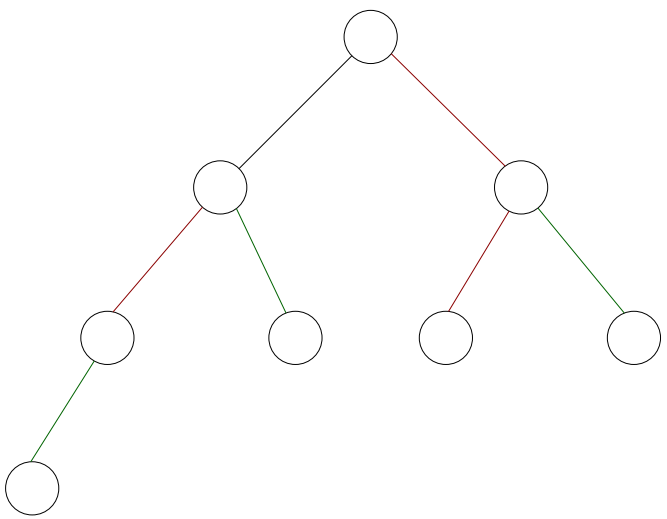
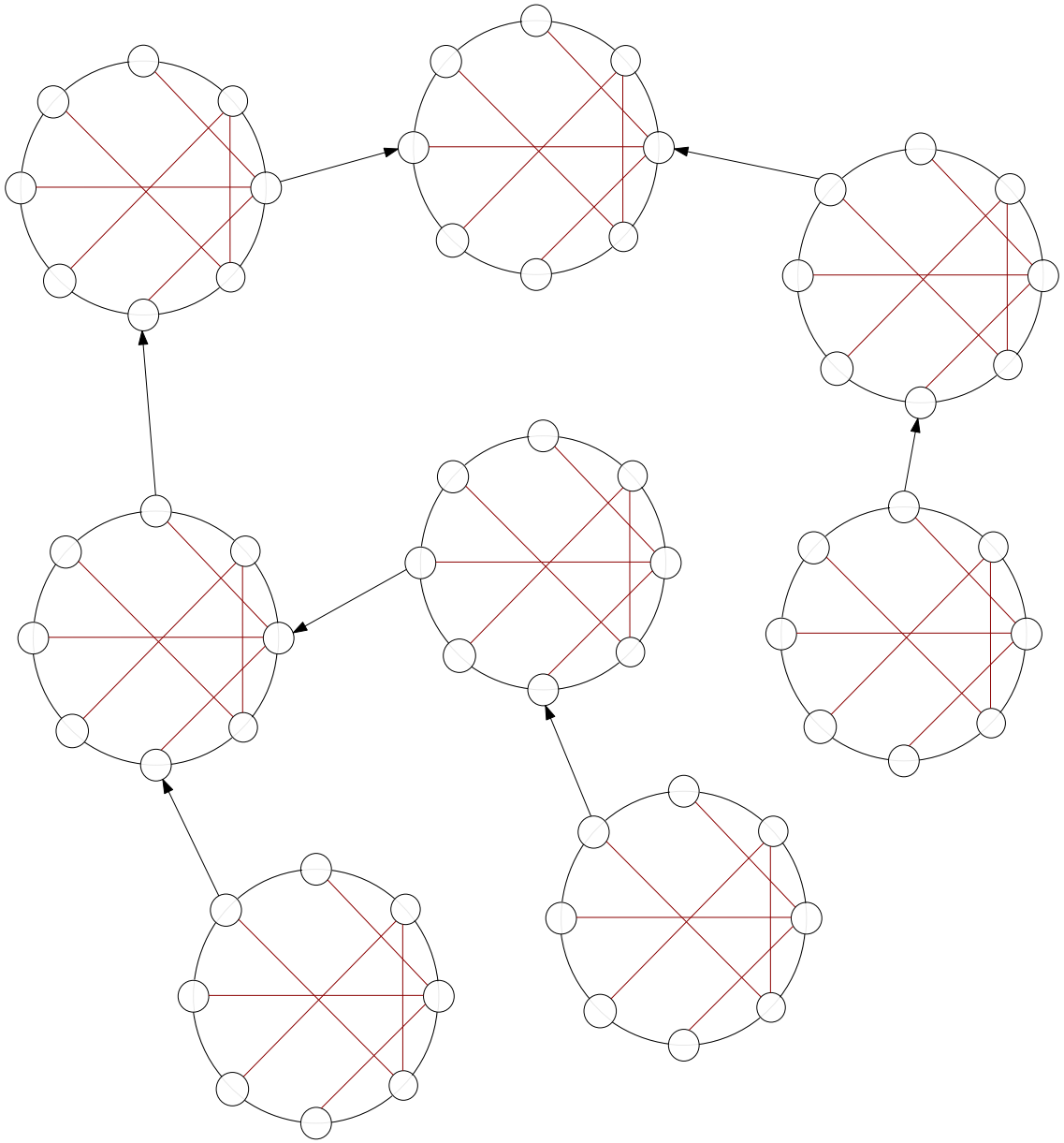
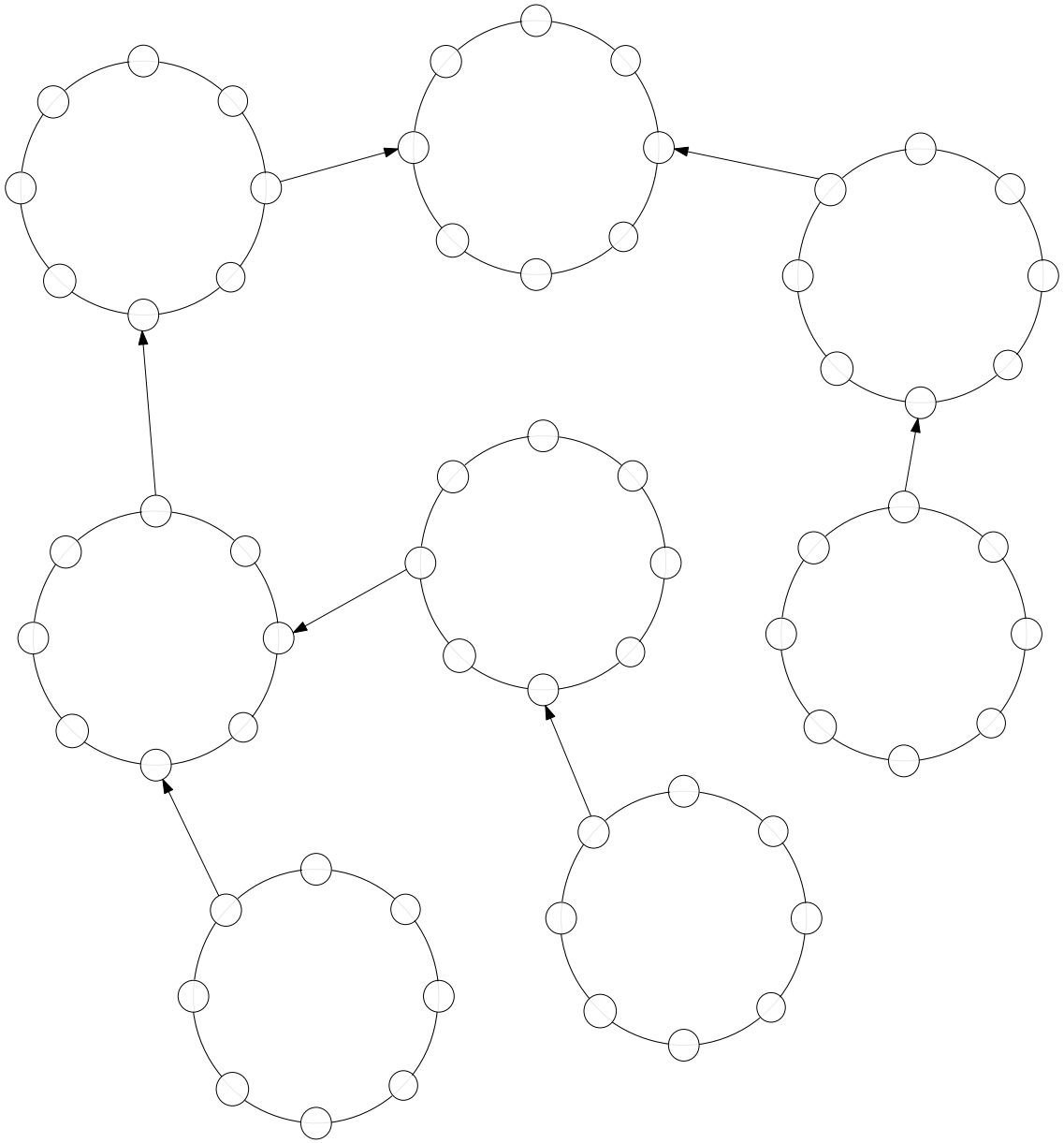






Low Level



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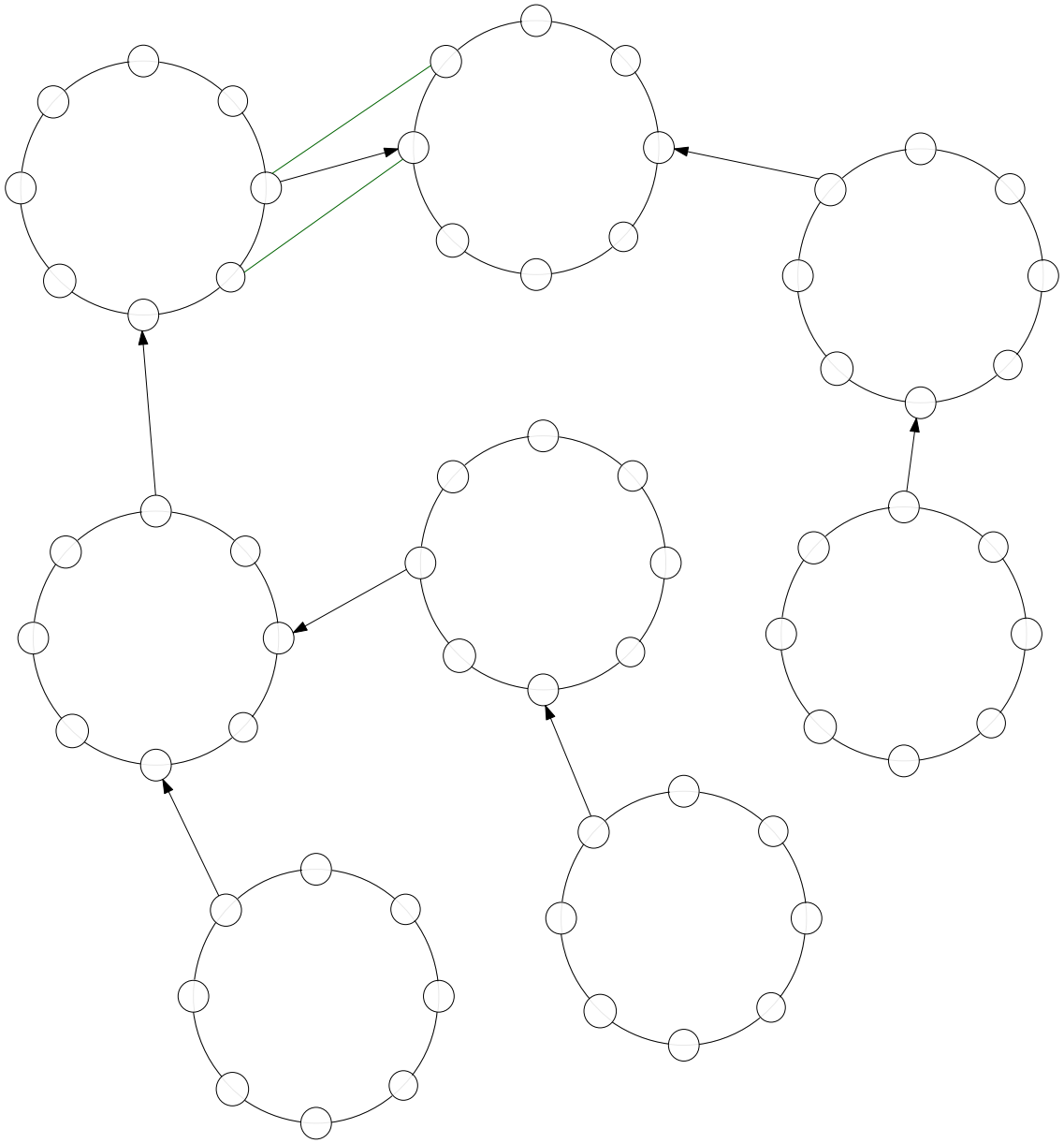

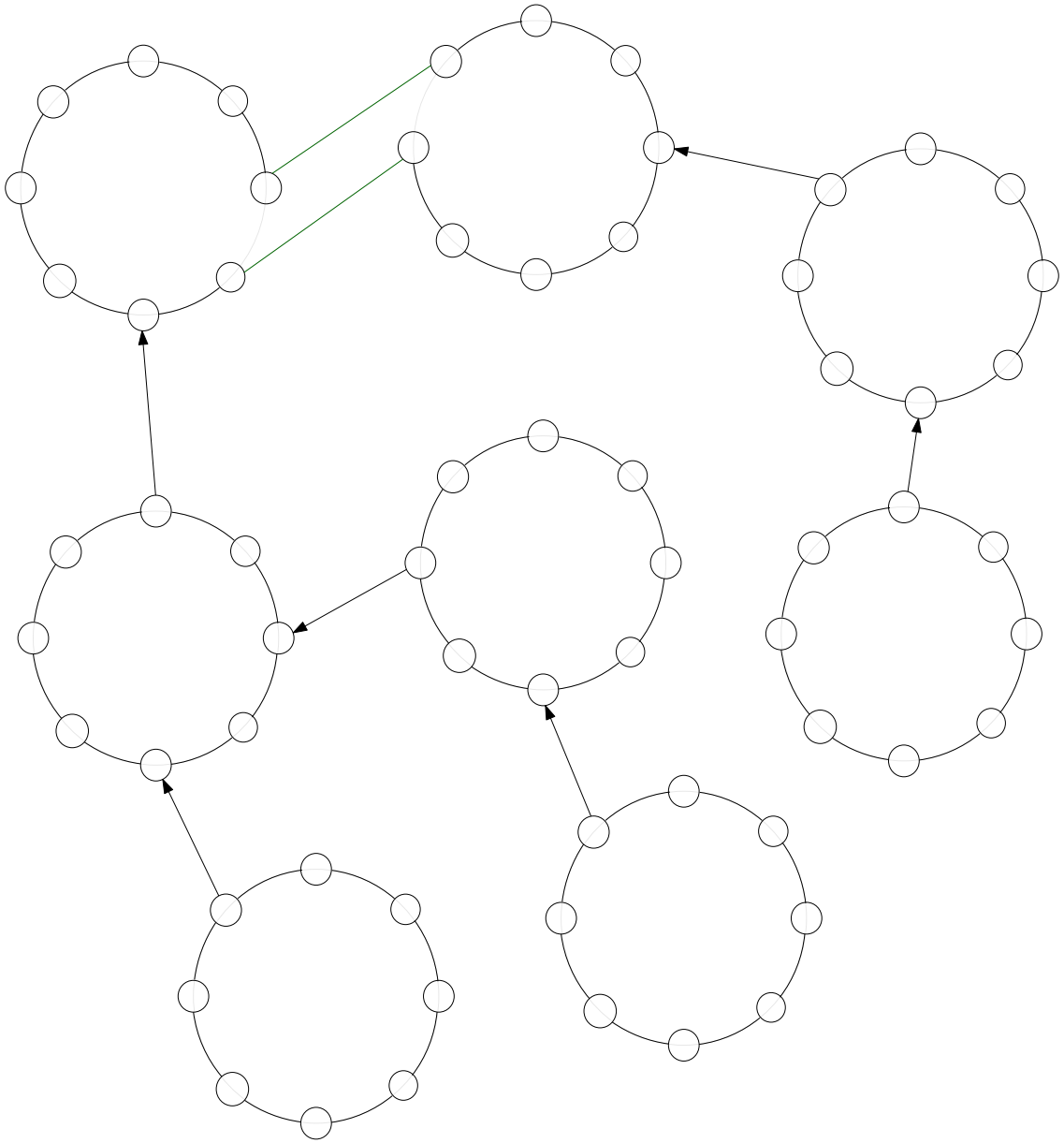

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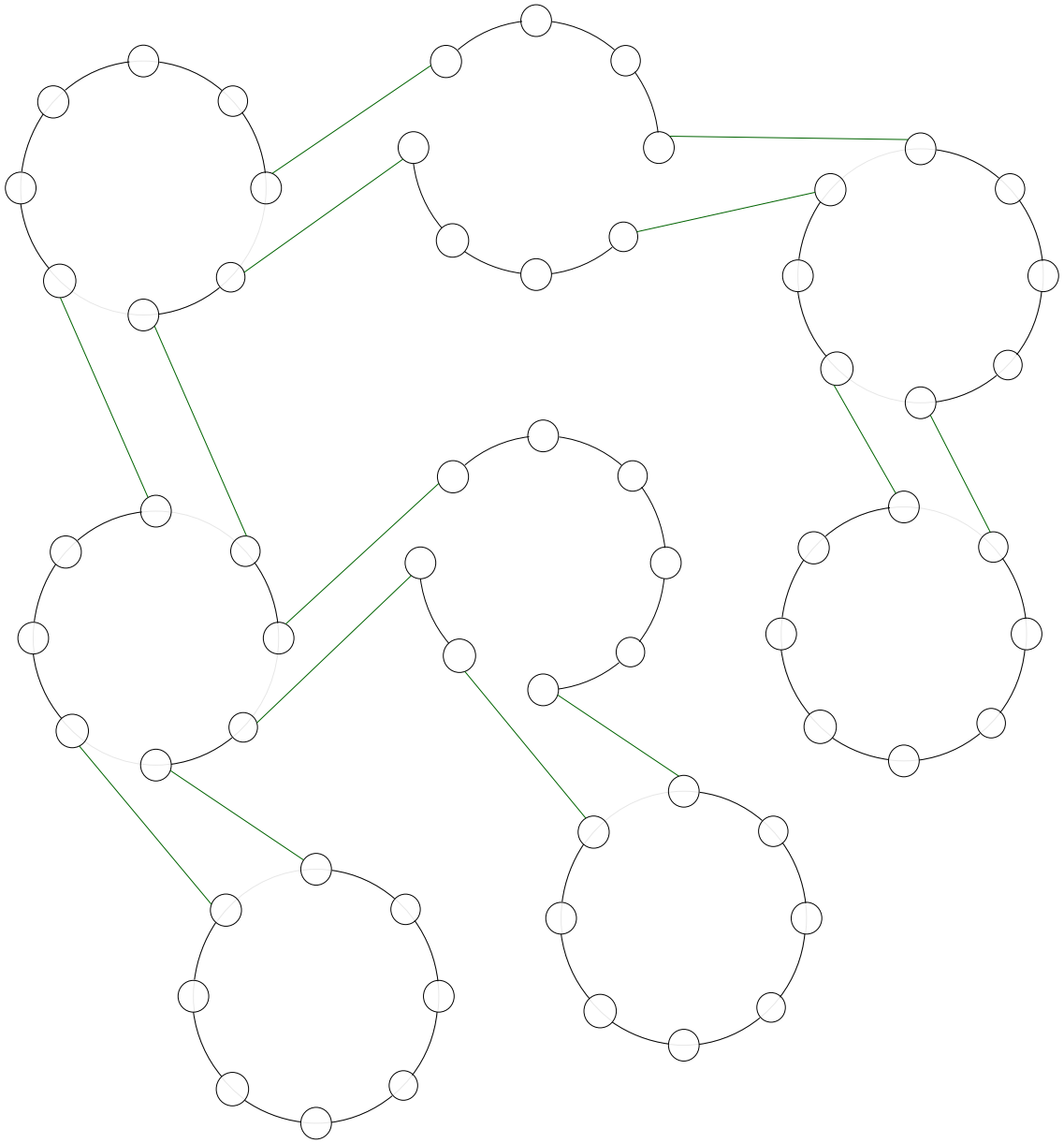
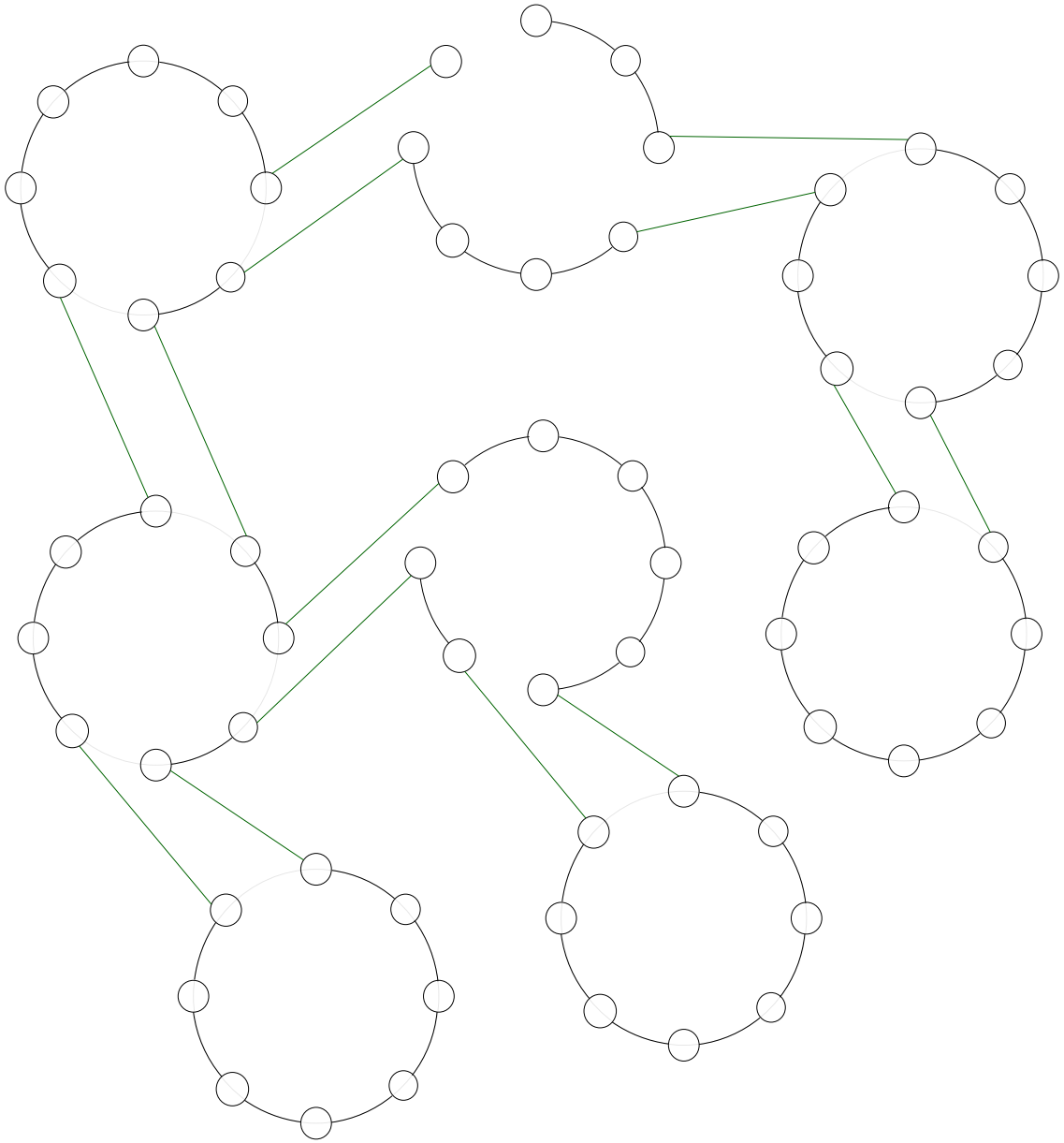


Low Level



Our Results

Algorithm	Time	Total Edge Activations	Maximum Activated Edges	Maximum Activated Degree
GraphToStar	$O(\log n)$	$O(n \log n)$	$O(n)$	$O(n)$
GraphToWreath	$O(\log^2 n)$	$O(n \log^2 n)$	$O(n)$	$O(1)$
GraphToThinWreath	$O(\log^2 n / \log \log n)$	$O(n \log^2 n)$	$O(n)$	$O(\log^2 n)$

Table: Algorithms for Depth-d Tree

Bounds	Time	Total Edge Activations	Maximum Activated Edges	Maximum Activated Degree
Centralized Lower	$\Omega(\log n)$	$\Omega(n)$	$\Omega(n/\log n)$	
Centralized Upper	$O(\log n)$	$\Theta(n)$		
Distributed Lower	$O(\log n)$	$\Omega(n \log n)$		

Table: Bounds for Depth-d Tree

- Motivation: Abstraction of networked systems which, starting from a single entity, can grow into well-defined global networks and structures
- Not well studied apart from the Nubot model of Woods *et al.* [WCG13], which is a geometric model

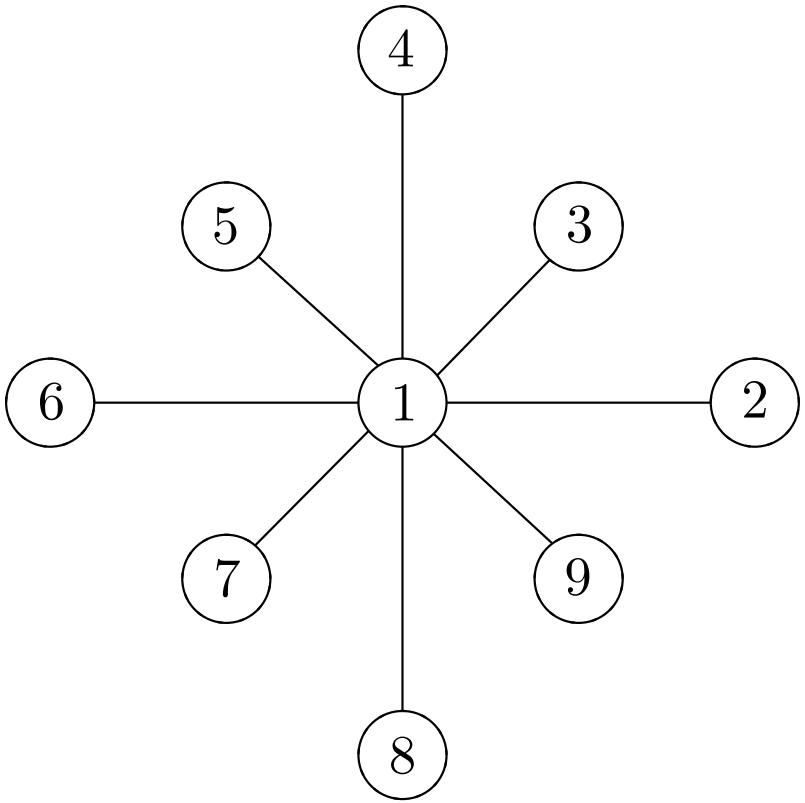
Our Focus

- Construct an input graph G starting from graph G_0 (graph with a single node) via node generations and edge activations
- Active Dynamics and Distributed Control
- BUT the centralized case is still unknown (and hard as it seems). So we will focus on the centralized case for now.

The Second Model

- Initial graph G_0
- Node generation: In every round, each node u can generate(give birth) another node v and activate edge uv
- Edge activation: At the time of its birth, node v can activate edges with nodes that are within its edge-activation distance d at the time of its birth
- Edge deletion. In every round, any node u can decide to delete some of its incident edges

An example



$a) 1 \to 2$

$b) 1 \to 4$

$c) 1 \to 6$

$d) 1 \to 8$

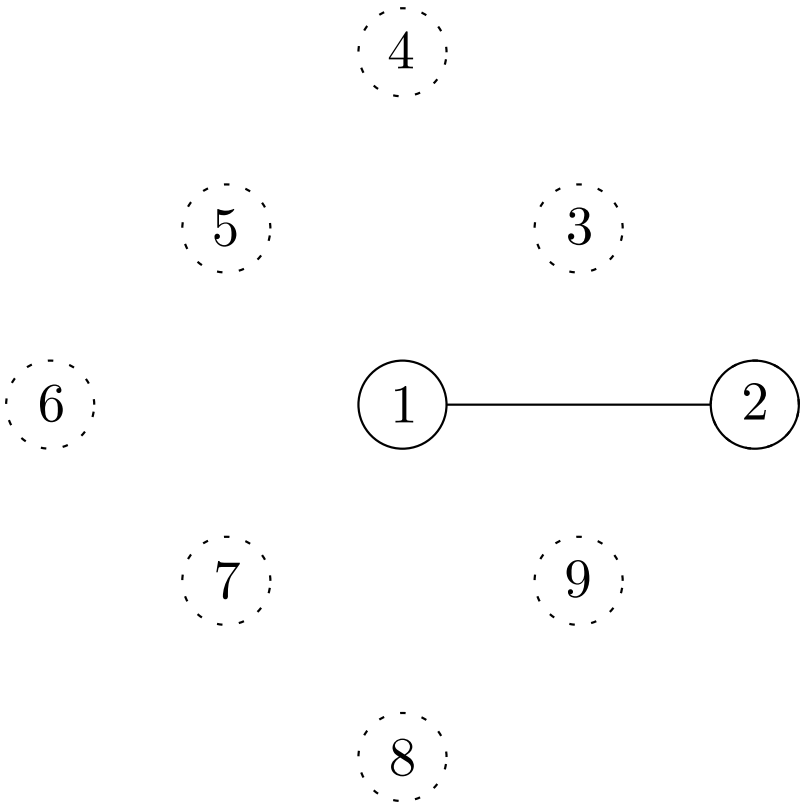
$e) 1 \to 9$

$f) 1 \to 3$

$g) 1 \to 5$

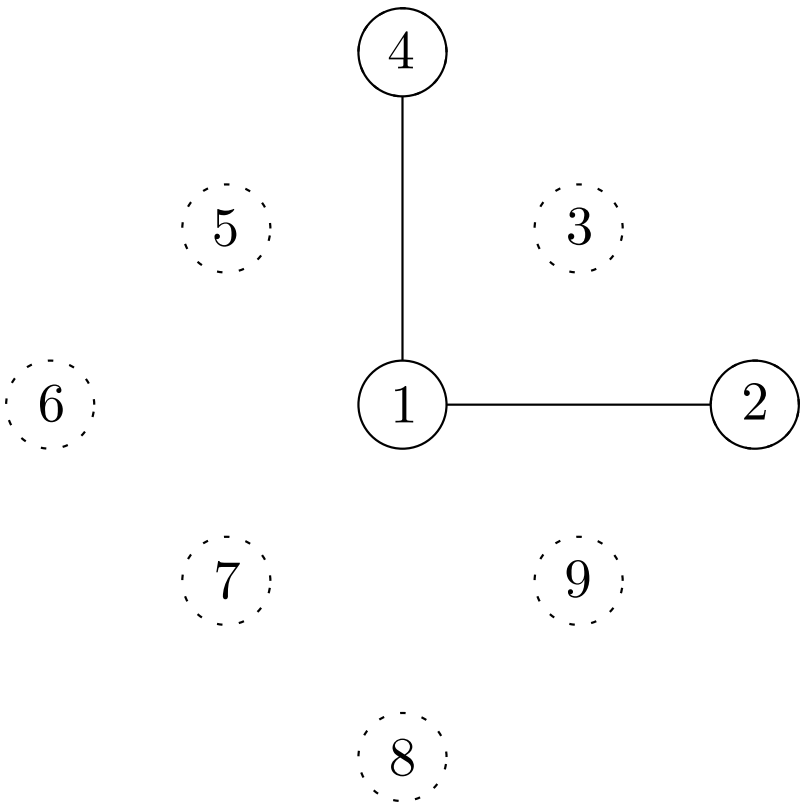
$h) 1 \to 7$

An example



$a) 1 \to 2$

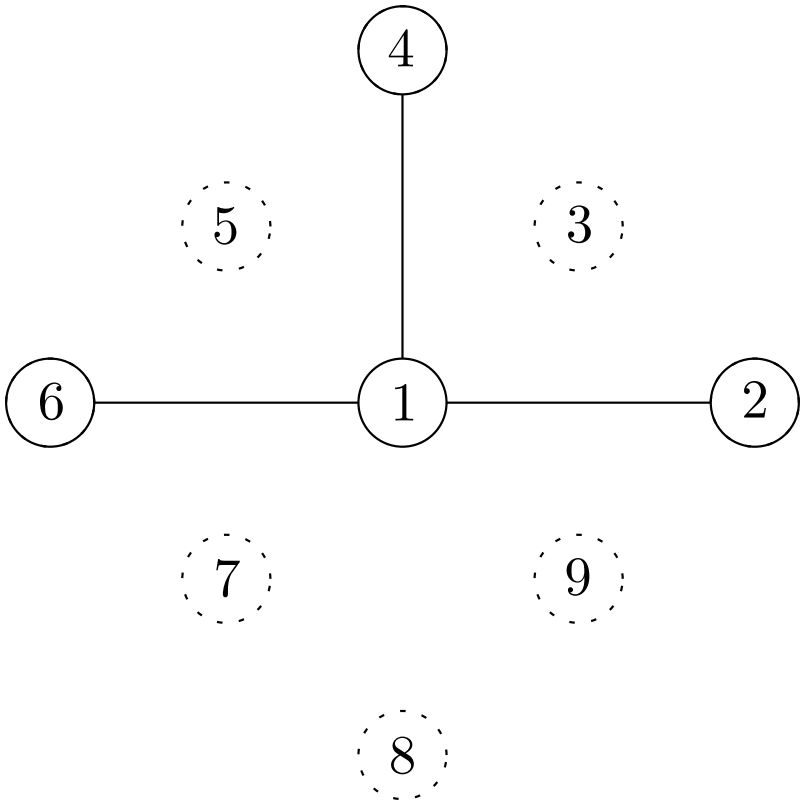
An example



$a)1 \to 2$

$b)1 \to 4$

An example



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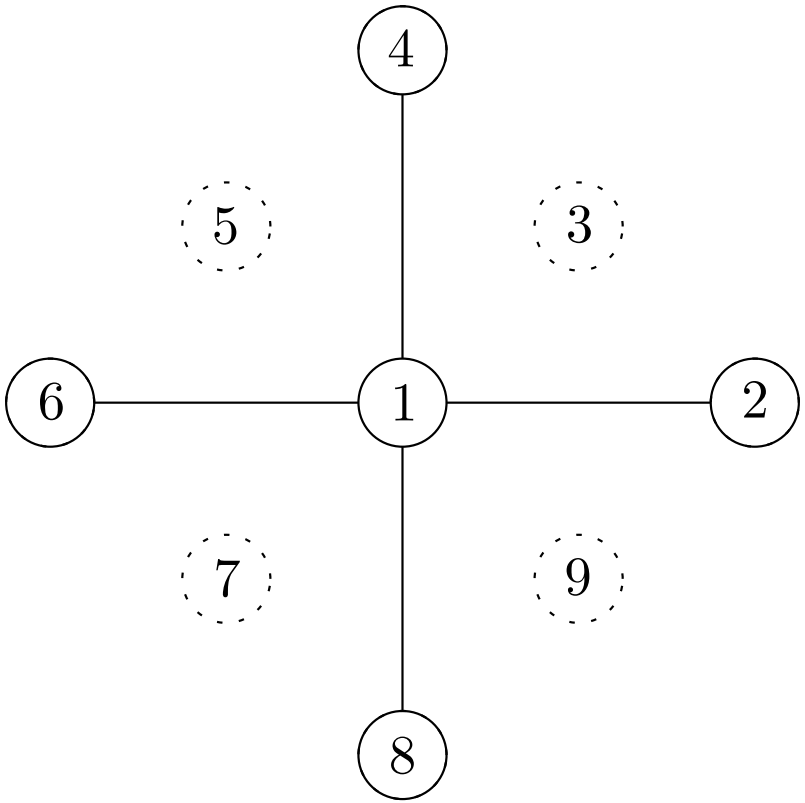

$a) 1 \to 2$

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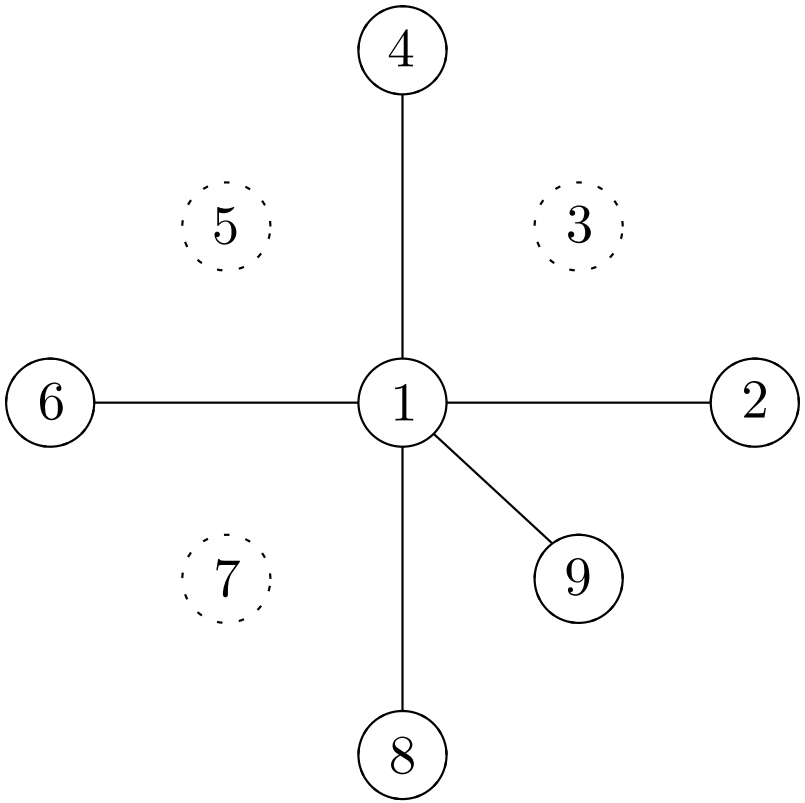
An example



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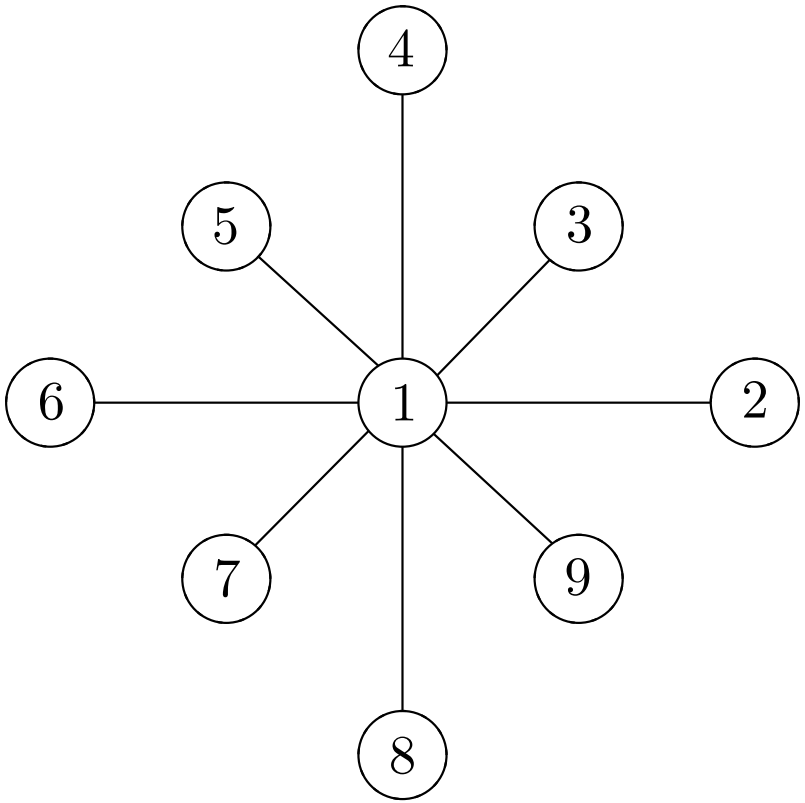

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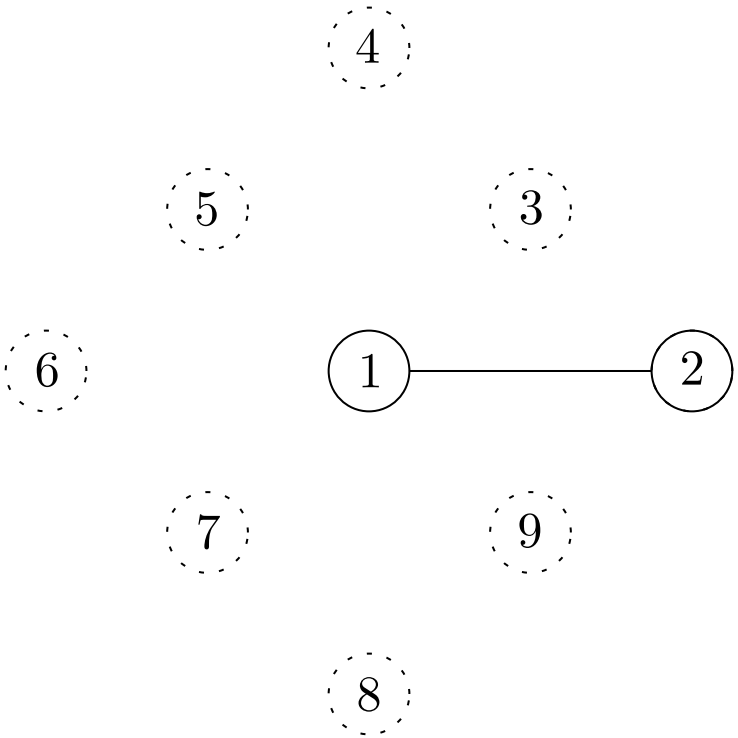
$f)1 \to 3$

$g)1 \to 5$

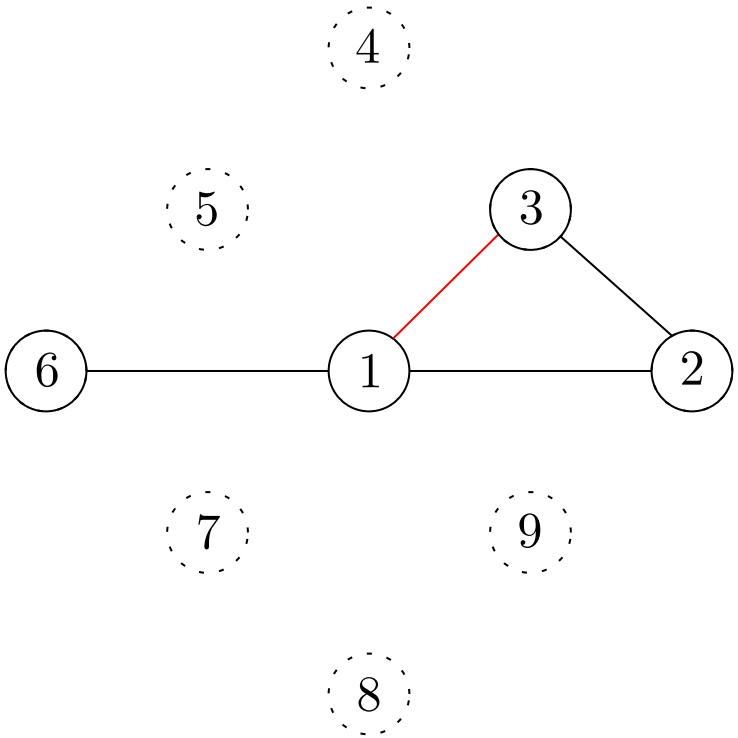
$h)1 \to 7$

An example



$a) 1 \to 2$

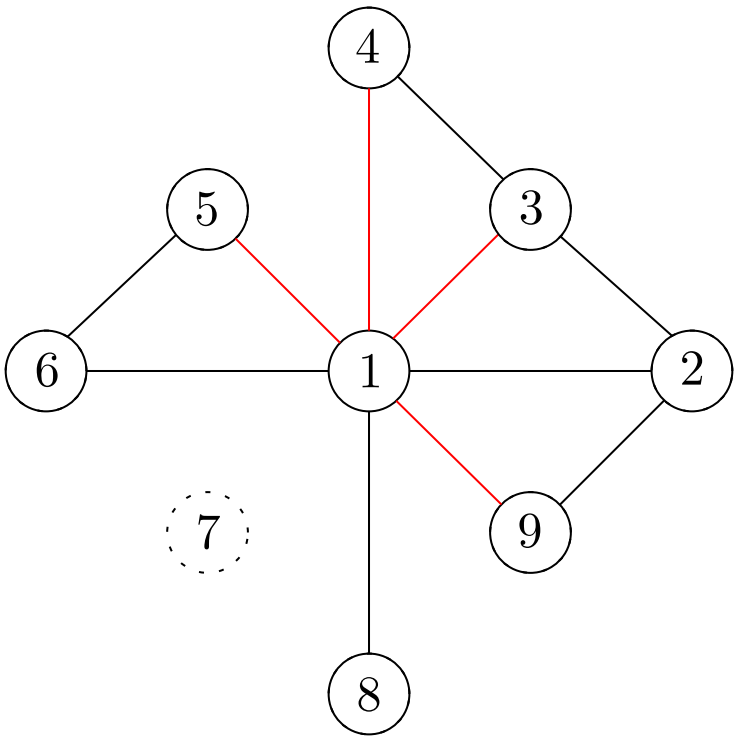
An example



$a) 1 \to 2$

$b) 1 \to 6, 2 \to 3, (1, 3)$

An example

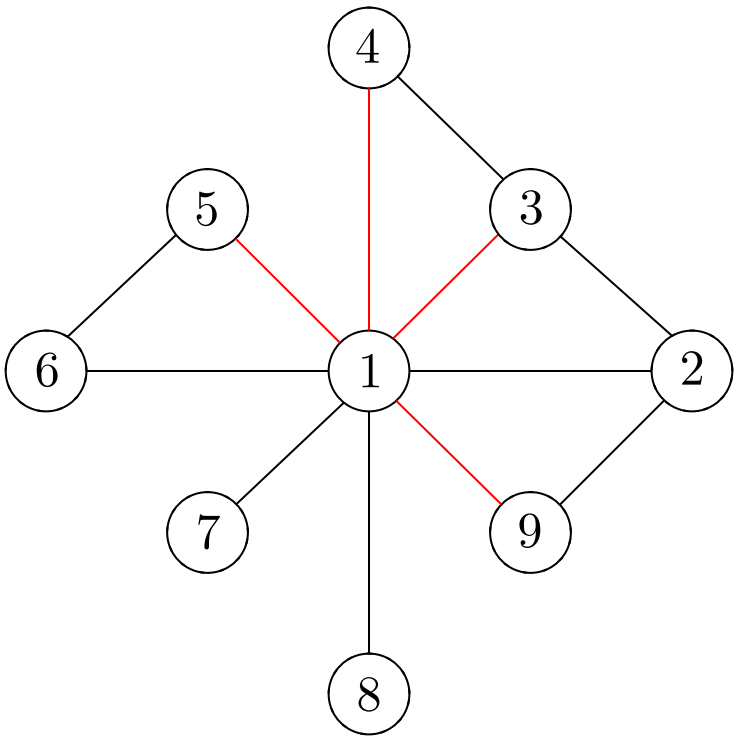


$a) 1 \to 2$

$b) 1 \to 6, 2 \to 3, (1, 3)$

$c) 1 \to 8, 2 \to 9, (1, 9), 3 \to 4, (1, 4), 6 \to 5, (1, 5)$

An example



$a) 1 \to 2$

$b) 1 \to 6, 2 \to 3, (1, 3)$

$c) 1 \to 8, 2 \to 9, (1, 9), 3 \to 4, (1, 4), 6 \to 5, (1, 5)$

$d) 1 \to 7, del((3, 4), (2, 3), (5, 6), (2, 9))$

The Problem

- Based on the previous example, we can see that there is a trade off between time slots and excess edges
- **Construction Schedule Problem**: Given an input graph G, compute in polynomial time a construction schedule of length at most k slots and with at most l excess edges, if it exists.
- **Zero Waste Construction Schedule Problem**: Given a input graph G, compute in polynomial time a construction schedule of length at most k slots and $l = 0$ excess edges, if it exists.

Edge Activation Distance

- Theorem: For $d = 1$, we provide an algorithm computes in polynomial time an optimal construction schedule with k slots for any tree graph G.
- Lemma: For $d \geq 4$, a graph $G = (V, E)$ with n nodes can be generated with a construction schedule with $\log n$ slots and with $O(n)$ excess edges.
- We will focus on $d = 2$ since its more natural and interesting

Properties:

- The nodes generated in a time slot form an independent set in the final graph.
- The progenies of two non adjacent nodes are independent from each other in the final graph.

Lower Bounds:

- Chromatic Number $\chi(G)$
- Size of largest clique c

Efficient Schedule for Trees

- The Efficient Schedule for Trees algorithm computes, in polynomial time, a construction schedule for any given tree graph G with $O(\log^2 n)$ slots and with $O(n)$ excess edges.
- Decomposition strategy where nodes are removed in phases until a single node is present
- The phases can be reversed using $O(\log^2 n)$ slots and $O(n)$ excess edges

Planar Graphs

- The Planar Graph algorithm computes, in polynomial time, a construction schedule for any given planar graph G with $O(\log n)$ slots and with $O(n \log n)$ excess edges.
- Compute a 5-coloring of the input planar graph
- Construct the nodes of each color class one by one using a line construction schedule for each color.
- $O(\log n)$ slots for each color class and $O(\log n)$ excess edges for each node.

Cop-win graphs: Undirected graph on which the pursuer (cop) can always win a pursuit-evasion game against a robber.

- A graph can be constructed with $l = 0$ iff it is a cop-win graph.
- The Recognition of Cop-win Graphs algorithm can decide in polynomial time, whether a given graph G is cop-win, and if so, it also produces a construction schedule with $n - 1$ slots and $l = 0$ excess edges.
- The Fast Cop-win algorithm computes in polynomial time a construction schedule σ for any graph $G = (V, E)$, where $|V| = 2^{\delta}$, with $\log n$ slots and $l = 0$ excess edges, if and only if such a σ exists for G.

Zero Waste Construction Schedule Problem

Theorem: The decision version of the zero-waste construction schedule problem is NP-complete.

- The reduction is from the coloring problem.
- Construct graph G' by adding a clique of size n whose nodes connect with every node in graph G.
- Show that the construction schedule for G' has $\kappa(G')$ slots such that $\kappa(G') = \chi(G) + n$.

Zero Waste Construction Schedule Problem

Theorem: Let $\epsilon > 0$. If there exists a polynomial-time algorithm, which, for every graph G, computes a $n^{1-\epsilon}$-approximate construction schedule, then P=NP.

- The reduction is from the coloring problem.
- Construct graph G'. Show that computing an approximate construction schedule for G' also computes an approximate solution for the chromatic number of G.

Open Problems

- Relationship to overlays (communication vs edges)
- Randomized algorithms, anonymous entities, less local edge activations, dynamic nodes
- Improve upper and/or lower bounds for both models
- Going back from networks to geometry (e.g., translating edge complexity to local moves)
- Distributed model for growing graphs

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Thank you!!!